ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 12, No. 2, 2017, *pp*.123-129

Image encryption based on the tracking control Hindmarsh-Rose system via Genesio-Tesi system

Keming Tang[1], Xuerong Shi [2*], Zuolei Wang [2], Wen Yun[2], Liying Zheng[2]
[1] *Academic Affairs Office, Yancheng Teachers University, Yancheng, 224002, China;*
[2] *School of Mathematics and Statistics, Yancheng Teachers University, Yancheng, 224002, China, E-mail: sxryctc@163.com.*
(Received December 22, 2016, accepted February 15, 2017)

Abstract. According to encryption structure of permutation and diffusion, as well as pseudo-randomness of chaotic synchronization sequences, tracking Hindmarsh-Rose system via Genesio-Tesi system is applied into the process of image encryption. Theoretical analysis and simulations suggest that the proposed encryption algorithm has good security in image transmission.

Keywords: tracking synchronization; Hindmarsh-Rose system; Genesio-Tesi system; image encryption.

1. Introduction

Chaos synchronization is making two chaotic systems identical after transient initial states. Since being proposed by Carroll and Pecora [1], it has been studied and applied in many fields, such as secure communication[2], biological systems[3], robotics[4]. It is found that synchronization is useful and has many potential applications in many domains[2-7], especially, synchronization in physical or biological system is a fascinating subject attracting many renewed attention [5].

In many kinds of synchronizations, one is about the synchronization between non-linear systems with different structures and orders, which can be seen as the variable states of the slave system are tracking the trajectories of the master system [8-12]. This problem can be transformed to a regulation problem with the origin (zero) as the corresponding set point. the trajectories of two systems will follow the same path after some transient. This synchronization needs weaker conditions to be realized than complete synchronization and it is can be regarded as partial synchronization [13, 14], which has been studied by many researchers in the past years[15-18].

In the last decades, due to the strong sensitivity to the initial value, chaos synchronization has been applied into digital image encryption[19-21] and good encryption results have been obtained.

In this paper, changeable gain coefficients are introduced into Lyapunov function to study tracking Hindmarsh-Rose system via Genesio-Tesi system. This tracking synchronization is applied into digital image encryption. Theoretical analysis and numerical simulations are presented to demonstrate the effectiveness of the proposed tracking scheme and the security of the encryption algorithm.

2. System descriptions

2.1. Hindmarsh-Rose system

Hindmarsh-Rose (HR) model, first proposed by Hindmarsh and Rose as a mathematical representation of the firing behavior of neurons, was originally introduced to give a bursting type with long inters pike intervals of real neurons [22], which is given as

$$\begin{aligned} \dot{x}_1 &= ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext}, \\ \dot{x}_2 &= c - dx_1^2 - x_2, \\ \dot{x}_3 &= r(S(x_1 + k) - x_3), \end{aligned} \tag{1}$$

where a, b, c, d, r, S, k, I_{ext} are real constants.

2.2. Genesio-Tesi system

The Genesio-Tesi system[23] consists of a simple square part and three simple differential equations depending on three positive real parameters. It can be written as

$$\dot{y}_1 = y_2,$$
$$\dot{y}_2 = y_3,$$
$$\dot{y}_3 = -a_1 y_1 - a_2 y_2 - a_3 y_3 + y_1^2, \tag{2}$$

where a_1, a_2, a_3 are positive real constants.

Next, tracking synchronization of Hindmarsh-Rose neuron system via Genesio -Tesi system will be discussed via a single controller.

3. Tracking synchronization of Hindmarsh-Rose system via Genesio-Tesi system

In this section, a scheme is proposed to realize the tracking synchronization of Hindmarsh-Rose neuron system via Genesio-Tesi system. This scheme needs only one single controller u, which is added to the second equation of system (2). The controlled Genesio-Tesi system is given as

$$\dot{y}_1 = y_2,$$
$$\dot{y}_2 = y_3 + u,$$
$$\dot{y}_3 = -a_1 y_1 - a_2 y_2 - a_3 y_3 + y_1^2. \tag{3}$$

To explore the tracking synchronization of Hindmarsh-Rose neuron system via Genesio-Tesi system, let the error be

$$e = x - y_1, \tag{4}$$

where x is one of the observed states of system (1). Lyapunov function is chosen as

$$V = \alpha e^2 + (\dot{e} + \beta e)^2, \tag{5}$$

where α and β are positive gain coefficients, the over dot denotes the differential variable e over time t. The differential of V is

$$\begin{aligned}\dot{V} &= 2\alpha e\dot{e} + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e}) \\ &= -2\beta V + 2\beta V + 2\alpha e\dot{e} + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e}) \\ &= -2\beta V + 2\beta[\alpha e^2 + (\dot{e} + \beta e)^2] + 2\alpha e\dot{e} + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e}) \\ &= -2\beta V + 2\alpha e(\dot{e} + \beta e) + 2\beta(\dot{e} + \beta e)^2 + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e}) \\ &= -2\beta V + 2(\ddot{e} + 2\beta\dot{e} + \alpha e + \beta^2 e)(\dot{e} + \beta e).\end{aligned} \tag{6}$$

When

$$2(\ddot{e} + 2\beta\dot{e} + \alpha e + \beta^2 e)(\dot{e} + \beta e) = 0 \tag{7}$$

is satisfied , we have

$$\frac{dV}{dt} = -2\beta V < 0. \tag{8}$$

It means that the errors of corresponding variables will be stabilized to a certain threshold. That is to say, the observed state x of system (1) and the salve state y_1 of the system (2) with a controller will reach synchronization.

To verify the feasibility of above method, three cases are to be considered and some numerical simulations will be demonstrated. In the numerical simulations, The system parameters are chosen as a = 3.0, b = 1.0, c = 1.0, d = 5.0, r = 0.006, S = 4.0, k = 1.6, I_{ext} = 3.0, a_1 = 6.0, a_2 = 2.88, a_3 =1.2. The initial conditions of the HR system and the Genesio-Tesi system are set as (0.1, 0.9, 0.8) and (0.4, 0.3, 0.2), respectively.

Case 1 Simulating the bursting activity of x_1 using y_1.

In this case, let $e_1 = x_1 - y_1$, the controller u is taken as

$$u_1 = (2ax_1 - 3bx_1^2)(ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext}) + (c - dx_1^2 - x_2)$$