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Finite-time burst synchronization of time-delay neural system with parameters disturbed by periodic signal

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Abstract. In this manuscript, finite-time burst synchronization of time-delay neuron system is investigated for two cases. In one case, parameters are known. In another case, parameters are unknown and some parameters are disturbed by periodic signal. The time to gain burst synchronization is derived and a factor affecting the synchronization time is given via theoretical analysis. The relationship between the time and the factor is described. Finally, simulation results are given to verify the effectiveness of the proposed methods.

Keywords: Finite-time burst synchronization; Hindmarsh-Rose system; Periodic perturbation; Time-delay

1. Introduction

In the past decades, synchronization of chaotic system has attracted more and more attention of researchers due to its powerful applications in many areas [1-3], such as secure communication, chemical reactions, biological systems and mechanical systems. Many kinds of synchronization have been investigated, such as complete synchronization [4], lag synchronization [5], generalized synchronization [6], phase synchronization [7], anti-synchronization [8], cluster synchronization [9], etc. For this, various effective methods have been proposed to synchronize chaotic systems, e.g., sliding mode control [10], back stepping method [11], adaptive control [12], observer-based control [13], nonlinear control [14], control Lyapunov function method [15], and so on.

With further research on synchronization, more and more people have realized that the time to achieve synchronization is very important in real applications. For this reason, many methods have been introduced to get faster convergence speed, among which finite-time control is an effective technique. Since finite-time synchronization means the optimality in convergence time, many contributions have been made to it [16-21]. Meanwhile, in real systems, time-delay is often inevitable. So it is necessary to investigate the finite-time synchronization of time delay system.

Motivated by potential applications of synchronization, many researchers have been engaged in the synchronization of neuron system. Studies [22-26] have shown that synchronization is an important phenomenon in information processing of neurons and neurons engage in various activities, among which, burst is one pattern consisting of the active phase and silent phase. Burst synchronization [27] naturally refers to the introduction of a temporal relationship between the bursts produced by two or more neurons. It is typically used to refer to a temporal relationship between active phase onset or offset times across neurons. This form of synchronization can be observed, when either excitatory synaptic coupling or diffusive coupling is introduced between a pair of model respiratory neurons [28, 29]. Clinical evidences suggest that burst synchronization plays an important role in some pathology, such as Parkinson's disease, essential tremor, and epilepsies [30]. Therefore, controlling this synchronization has a practical importance for undesirable neuronal rhythms [31, 32]. In addition, in real neuron system, time-delay always exists when signals are communicated among neurons, even in the same neuron. So it is necessary to investigate the finite-time burst synchronization of the neuron system with time-delay.

Based on above, the main contribution of this paper is to investigate the finite-time burst synchronization of time-delay neuron system with various parameters. The rest parts of this paper are arranged as follows. Some preliminaries are given in Section 2. In Section 3, the finite-time burst synchronization of time-delay neuron system is discussed for two cases. One is with known parameters and another is with periodic perturbation parameters. Section 4 gives numerical simulations to verify the effectiveness of the proposed method. Some conclusions are reached in Section 5.

2. Preliminaries

Definition 1. Considering two chaotic systems as follows:

$$\dot{x} = f(x), \dot{y} = g(y) \tag{1}$$

Where x,y are two n-dimensional state vectors. $f, g : R^n \to R^n$ are vector-valued functions. If there exists a positive constant T such that

$$\lim_{t\to T} \|x-y\| = 0,$$

and $\|x-y\| \equiv 0$ when $t \geq T$, then it is said that the two systems of (1) can achieve finite-time synchronization.

Lemma 1[33]. Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality

$$\dot{V}(t) \leq -cV^{\eta}(t), \forall t \geq t_0, V(t_0) \geq 0, \tag{2}$$

where $c > 0$, $0 < \eta < 1$ are all constants. Then for any given t_0, $V(t)$ satisfies following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0), t_0 \leq t \leq t_1, \tag{3}$$

and

$$V(t) \equiv 0, \ \forall t \geq t_1 \tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

Lemma 2 [34]. Suppose $0 < r \leq 1$, a, b are all positive numbers, then the inequality

$$(a+b)^r \leq a^r + b^r$$

is quite straightforward .

3. Finite-time burst synchronization of neuron system with time-delay

3.1 System description

In this section, the neuron system with time-delay is considered as following Hindmarsh -Rose (HR) system:

$$\dot{x} = ax^2 - bx^3 + y - z(t-\tau) + I_{ext},$$

$$\dot{y} = c - dx^2 - y,$$

$$\dot{z} = r(S(x+k) - z), \tag{6}$$

where $\tau > 0$ is the time-delay. x, y and z represent the membrane potential of the neuron, the recovery variable, and the adaptation current, respectively. a, b, c, d, r, S, k are real constants. I_{ext} is an external influence on the system. When τ =0, model (6) is a mathematical representation of the firing behavior of neurons proposed by Hindmarsh and Rose [35]. In Eq. (6), time delay emerges in the third variable z, which is thought as adaptation current. The electric synaptic exists in many neurons, and its effect is often described by an additional current with time delay [36, 37]. With various I_{ext}, model (6) can show different dynamical behaviors. For example, if $\tau = 1$ and other parameters are chosen as a =3.0, b =1.0, c =1.0, d =5.0, r =0.006, S =4.0, k =1.6, system (6) can show regular bursting for $I_{ext} = 2.6$ (Fig.1) and chaotic bursting for $I_{ext} = 3.1$ (Fig.2), respectively.