ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 12, No. 3, 2017, *pp.*163-182

Notes on Fuzzy Linear Systems

Sanhita Banerjee*, Tapan Kumar Roy
Indian Institute of Engineering Science and Technology, Shibpur, (formerly BESU, Shibpur)
Howrah-711103, West Bengal, India
E-mail: sanhita.banerjee88@gmail.com.
(Received March 20, 2017, accepted May 11, 2017)

Abstract. The paper discusses the detail solution procedure of fuzzy linear systems and fully fuzzy linear system. Each case has been solved by using the concept of Strong and Weak solution with some numerical examples. We have also developed some theorems regarding the existence of the solution.

Keywords: Fuzzy Linear System (FLS), Fully Fuzzy Linear System(FFLS), Generalized Trapezoidal Fuzzy Number (GTrFN), Strong and Weak solutions.

1. Introduction

System of linear equations has so many applications in various areas of mathematical, physical and engineering sciences. It is used to solve several problems in fluid flow, circuit analysis, heat transport, structural mechanics etc. In most of the problems, the measurements and system's parameters are vague or imprecise. We can handle the situation by representing the given data as the fuzzy numbers rather than crisp numbers.

In literature, the concept of fuzzy numbers and arithmetic operations on it introduced by Zadeh [10,11]. Further standard analytical techniques to solve fuzzy linear equation and linear system were proposed by Buckley and Qu [5,6,8,9]. Buckley [6,7] considered the solution of linear fuzzy equations using Classical methods and Zadeh's extension principle. Fuzzy linear systems are the linear systems whose parameters are all or partially represented by fuzzy numbers. A general model for solving a Fuzzy linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number was first proposed by Friedman et al. [12]. They have used the parametric form of fuzzy numbers and replace the original n×n fuzzy system by a 2n×2n crisp system. Fuzzy linear system has been studied by several authors [1,2,4,13,14,15, 16, 17,18].

In this paper we have used another approach to solve Fuzzy Linear System and Fully Fuzzy Linear System. Here we have solved both systems respectively by using the concept of Strong and Weak solution .We have also developed some conditions following[12] for existence of the strong solution. Each case has been illustrated by numerical examples along the methods.

2. Basic Concept

Definition 2.1: Fuzzy Set: Let X be a universal set. The fuzzy set $\tilde{A} \subseteq X$ is defined by the set of tuples as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \colon \mu_{\tilde{A}} \colon X \to [0,1]\}$.

Definition 2.2: ∝-cut of a fuzzy set: Let X be an universal set. Let $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\} (\subseteq X)$ be a fuzzy set. ∝-cut of the fuzzy set $\tilde{A}$ is a crisp set. It is denoted by $A_\propto$. It is defined as

$$A_\propto = \{x : \mu_{\tilde{A}}(x) \geq \propto \ \forall x \in X\}.$$

Definition 2.3: Convex fuzzy set: A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\} \subseteq X$ is called convex fuzzy set if all $A_\propto$ are convex sets i.e. for every element $x_1 \in A_\propto$ and $x_2 \in A_\propto$ and for every $\propto \in [0,1]$
$\lambda x_1 + (1-\lambda)x_2 \in A_\propto \quad \forall \lambda \in [0,1]$. Otherwise the fuzzy set is called non convex fuzzy set.

Definition 2.4: Fuzzy Number: $\tilde{A} \in \mathcal{F}(R)$ is called a fuzzy number where R denotes the set of whole real numbers if

i. $\tilde{A}$ is normal i.e. $x_0 \in R$ exists such that $\mu_{\tilde{A}}(x_0) = 1$.
ii. $\forall \propto \in (0,1]$ $A_\propto$ is a closed interval.

If $\tilde{A}$ is a fuzzy number then $\tilde{A}$ is a convex fuzzy set and if $\mu_{\tilde{A}}(x_0) = 1$ then $\mu_{\tilde{A}}(x)$ is non decreasing for $x \leq x_0$ and non increasing for $x \geq x_0$.

The membership function of a fuzzy number $\tilde{A}\ (a_1, a_2, a_3, a_4)$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x \in [a_2, a_3] \neq \phi \\ L(x), & a_1 \leq x \leq a_2 \\ R(x), & a_3 \leq x \leq a_4 \end{cases}$$

Where L(x) denotes an increasing function and $0 < L(x) \leq 1$ and R(x) denotes a decreasing function and $0 \leq R(x) < 1$.

Definition 2.5: Generalized Fuzzy Number (GFN): Generalized Fuzzy number $\tilde{A}$ as

$$\tilde{A} = (a_1, a_2, a_3, a_4; w),$$

where

$$0 < w \leq 1,$$

and a_1, a_2, a_3, a_4 $(a_1 < a_2 < a_3 < a_4)$ are real numbers. The Generalized Fuzzy Number $\tilde{A}$ is a fuzzy subset of real line R, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

1) $\mu_{\tilde{A}}(x)$: R → [0, 1]
2) $\mu_{\tilde{A}}(x) = 0$ for $x \leq a_1$
3) $\mu_{\tilde{A}}(x)$is strictly increasing function for $a_1 \leq x \leq a_2$
4) $\mu_{\tilde{A}}(x) = w$ for $a_2 \leq x \leq a_3$
5) $\mu_{\tilde{A}}(x)$is strictly decreasing function for $a_3 \leq x \leq a_4$
6) $\mu_{\tilde{A}}(x) = 0$ for $a_4 \leq x$

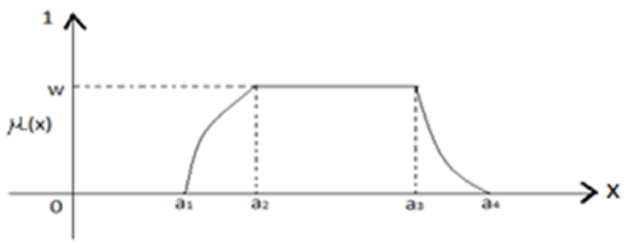

Fig-2.1:-Membership function of a GFN

Definition2.6: Generalized Trapezoidal Fuzzy number (GTrFN):

A Generalized Fuzzy Number $\tilde{A} = (a_1, a_2, a_3, a_4; w)$, is called a Generalized Trapezoidal Fuzzy Number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a_1 \\ w\dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ w, & a_2 \leq x \leq a_3 \\ w\dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & x \geq a_4 \end{cases}$$

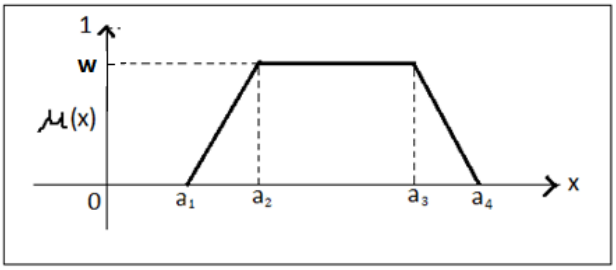

Fig-2.2:-Generalized Trapezoidal Fuzzy Number (GTrFN)

Here $l_s = a_2 - a_1$ is called left spread and $r_s = a_4 - a_3$ is called right spread.