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Daubechies wavelet based full approximation scheme for solving Burgers' equation arising in Fluid Dynamics

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Abstract. This paper presents, Daubechies wavelet based full approximation scheme (DWFAS) for the numerical solution of Burgers' equation, which is nonlinear partial differential equation (PDE) arising in fluid dynamics using Daubechies wavelet intergrid operartors. The numerical solutions obtained are compared with existing numerical methods and exact solution. Some of the test problems are presented to demonstrate that DWFAS has fast convergence in low computational time and is very effective, convenient and quite accurate to systems of PDEs.

Keywords: Daubechies wavelet; Multi-resolution analysis; Full approximation scheme; Burgers' equation.

1. Introduction

Burgers' equation has attracted much attention. Solving this equation has been an interesting task for mathematicians. This equation has been found to describe various kinds of phenomena such as a mathematical model of turbulence and approximate theory of flow through a shock wave traveling in a viscous fluid [1]. Consider one–dimensional non-linear PDE with the following initial and boundary conditions:

$$\frac{\partial u(x,t)}{\partial t} + u(x,t)\frac{\partial u(x,t)}{\partial x} = \upsilon \frac{\partial^2 (x,t)}{\partial x^2} \tag{1.1}$$

Initial condition:

$$u(x\,,0) = f(x), \quad 0 \le x \le 1 \tag{1.2}$$

Boundary conditions:

$$u(0\,,t) = g(t), \quad u(1\,,t) = h(t), \; t > 0 \tag{1.3}$$

is known as Burgers' equation. Burgers' model of turbulence is a very important fluid dynamics model and the study of this model and the theory of shock waves have been considered by many authors both for conceptual understanding of a class of physical flows and for testing various numerical methods. The distinctive feature of Eq. (1.1) is that it is the simplest mathematical formulation of the competition between non-linear advection and the viscous diffusion. It contains the simplest form of non-linear advection term and dissipation term where ' ' is the viscosity coefficient for formulating the physical phenomena of wave motion and thus determines the behavior of the solution. In 1915, such type of equation is introduced by Bateman [2] and proposed the steady-state solution of the problem. Burgers [3] introduced this equation in 1948, to capture some features of turbulent fluid in a channel caused by the interaction of the opposite effects of convection and diffusion, therefore it is referred as ''Burgers' equation''. The structure of Burgers' equation is roughly similar to that of Navier–Stokes equations due to the presence of the non-linear convection term and the occurrence of the diffusion term with viscosity coefficient. So, this equation can be considered as a simplified form of the Navier–Stokes equations and also it is the simplest model of nonlinear partial differential equation for diffusive waves in fluid dynamics. The study of the general properties of Burgers' equation has attracted attention of scientific community due to its applications in many physical problems including one-dimensional sound/shock waves in a viscous medium, waves in fluid filled viscous elastic tubes, magneto hydrodynamic waves in a medium with finite electrical conductivity, mathematical modeling of turbulent fluid, and in continuous stochastic processes.

Analytical methods for solving Burger's equation are very restricted and can be used in very special cases; so they cannot be used to solve equations of numerous realistic scenarios. Numerical methods which are commonly used such as finite difference, finite element methods etc. are need a large amount of computation and usually the effect of round-off error causes the loss of accuracy.

So far many authors are applied various numerical methods to solve Burgers equations, some of them are finite element method [4], Least-squares quadratic B-spline finite element method [5], Cubic B-splines collocation method [6] etc. For large systems, these methods are inefficient in terms of both computer storage and computational cost.

The multigrd approach is an alternative scheme to overcome these drawbacks. In 1964 Fedorenko [7] formulated a multigrid scheme to solve the Poisson equation in a recatangular domain. Bachvalov [8] generalized the technique for general elliptic PDEs in 1966. Up to this time, the approch was not yet practical. In 1973 the first practical results were published in a pioneering paper by Brandt [9]. He outlined the purpose and practical utility of multigrid methods. Hackbush [10] independently discovered multigrid methods and provided some theoretical foundation in 1976. The multigrid method is largely applicable in increasing the efficiency of iterative methods used to solve large system of algebraic equations. Since their early application to elliptic partial differential equations, multigrid methods have been applied successfully to a large and growing class of problems. Classical multigrid begins with a two-grid process. First, iterative relaxation is applied, whose effect is to smooth the error. Then a coarse-grid correction is applied, in which the smooth error is determined on a coarser grid. This error is interpolated to the fine grid and used to correct the fine-grid approximation. Applying this method recursively to solve the coarse-grid problem leads to multigrid.

Bastian et al. [11] was investigated in series of experiments to solve parabolic PDEs using multigrid methods. However, when meet by certain problems, for example parabolic type of problems with discontinuous or highly oscillatory coefficients, as well as advection-dominated problems, the standard multigrid procedure converges slowly with larger computational time or may break down. For this reason we go for wavelet multigrid method in which by choosing the filter operators obtained from wavelets to define the prolongation and restriction operators.

"Wavelets" have been very popular topic of conversations in many scientific and engineering gatherings these days. Some of the researchers have decided that, wavelets as a new basis for representing functions, as a technique for time-frequency analysis, and as a new mathematical subject. Of course, "wavelets" is a versatile tool with very rich mathematical content and great potential for applications. However wavelet analysis is a numerical concept which allows one to represent a function in terms of a set of basis functions, called wavelets, which are localized both in location and scale. In wavelet applications to the solution of partial differential equations the most frequently used wavelets are those with compact support introduced by Daubechies [12]. Recently, many authors De Leon [13], Bujurke et al. [14] and Shiralashetti et al. [15] have developed wavelet multigrid methods.

This paper gives an alternative method i.e. Daubechies Wavelet based full approximation scheme for the numerical solution of Burgers equation using Daubechies filter coefficients. Daubechies FAS is formulated in this paper have the following characteristics:

- Provide approximations which are continuous and continuously differentiable throughout the domain of the problems, and have piecewise continuous second derivatives.
- The methods possess super convergence properties.
- The methods incorporate IC and BCs in a systematic fashion.

The organization of the paper is as follows. Preliminaries of Daubechies wavelets are given in section 2. Section 3 deals with Wavelet multigrid operators. Method of solution is discussed in section 4. Numerical findings and error analysis are presented in section 5. Finally, conclusions of the proposed work are discussed in section 6.

2. Preliminaries of Daubechies wavelets

A major problem in the development of wavelets during the 1980s was the search for a multiresolution analysis where the scaling function was compactly supported and continuous. As we know, the Haar multiresolution analysis is generated by a compactly supported scaling function that is not continuous. The