

# Complete synchronization of 4D Chua system via linear controllers

Xuerong Shi<sup>1\*</sup>, Zuolei Wang<sup>1</sup>, Wen Yun<sup>1</sup>, Liying Zheng<sup>1</sup>

<sup>1</sup> School of Mathematics and Statistics, Yancheng Teachers University, Yancheng 224002, China

E-mail: sxryctc@163.com.

(Received February 17, 2017, accepted April 09, 2017)

**Abstract.** In this paper, the complete synchronization of 4D Chua systems is investigated via linear controllers. Especially, the synchronization can be realized using only one or two linear controllers. By analyzing simulation results, it is known that when realizing the synchronization, the choice of linear controller has greater flexibility and the controlled variables can also be randomly selected.

**Keywords:** Complete synchronization, hyperchaotic Chua system, linear controller

## 1. Introduction

Since being proposed by Pecora and Carroll [1], synchronization of chaotic systems has been paid much attention by many researchers[2-6]. Many kinds of methods have been discussed to study synchronization of chaotic systems, such as a stable-manifold-based method[7], adaptive method[8], back stepping scheme[9], sliding mode control[10], nonlinear control[11], and so on. Correspondingly, different types of synchronization have been proposed, for example, complete synchronization[12], anti-synchronization[13], phase synchronization[14], hybrid synchronization [15], etc.

With further research on the synchronization, many chaotic systems have been explored, for instance, Lorenz system, Chen system, Rössler system, Chua system, etc. In these systems, at least one nonlinear term is involved in addition to linear terms. Chua system is a simple non-linear electronic circuits with chaotic behaviors. Initially, Leon. O. Chua addressed a 3D Chua's circuit, which becomes a typical model for studying chaos due to its simplicity and representation of the nonlinear circuit. Chua system attracted researchers' attention and has been used widely. Later, some researchers found that 3D Chua system can not satisfied the requirements of studying circuit system and a 4D Chua system was put forward[16]. Afterwards, the dynamics of the 4D Chua system has been extensively studied.

Based on above, complete synchronization of a 4D Chua system is investigated via linear controllers in this paper. Other parts of this paper are arranged as follows. Section 2 depicts the 4D Chua system and the attractors of it. In Section 3, the complete synchronization of the 4D Chua system is discussed via linear controllers using numerical simulations. Conclusions are drawn in Section 4.

## 2. Model description

In this section, hyperchaotic Chua system is considered as:

$$\begin{aligned}\dot{x}_1 &= a(x_2 - px_1 - qx_1^3), \\ \dot{x}_2 &= x_1 - x_2 + x_3 + x_4, \\ \dot{x}_3 &= -bx_2 + x_4, \\ \dot{x}_4 &= -\gamma x_1 + \rho x_2 + \omega x_4.\end{aligned}\tag{1}$$

If the parameters are chosen as  $b=16$ ,  $\gamma=0.1$ ,  $\rho=0.6$ ,  $\omega=-0.03$ ,  $p=-1/7$  and  $q=6/7$ , the dynamical behaviors of system (1) shows diversity for different values of  $a$ , which can be verified by simulation results. In our numerical calculations, the fourth order Runge-kutta algorithm is used, the time step is  $h=0.01$ , and the initial values for the variables of model (1) are selected as (0.01, 0.02, 0.03, 0.04), (-0.6, -0.5, -0.9, -0.8), (0.6, 0.5, 0.9, 0.8), (-0.01, -0.02, -0.03, -0.04), respectively.

The phase portraits of system (1) are plotted in Figs.1-4. When  $a=7.5$ , system (1) has only one periodical attractor for different initial values (Fig.1). But for  $a=7.745$  and  $a=8.6$ , system (1) performs

coexistence of two periodical attractors (Fig.2) and three periodical attractors (Fig.3), respectively. When  $a=8.9$ , two periodical attractors and one chaotic attractor can coexist in system (1) (Fig.4).

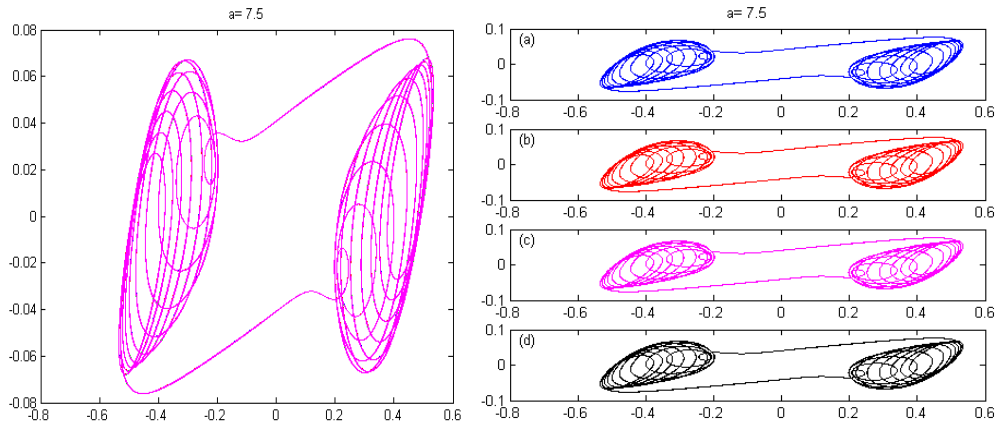


Fig.1 Periodical attractor on the platform of  $(x, y)$  of system (1) when  $a=7.5$ , the right part is the cases for different initial values of system (1), (a) (0.01,0.02,0.03,0.04), (b) (-0.6,-0.5,-0.9,-0.8), (c) (0.6,0.5,0.9,0.8), (d) (-0.01,-0.02,-0.03,-0.04).

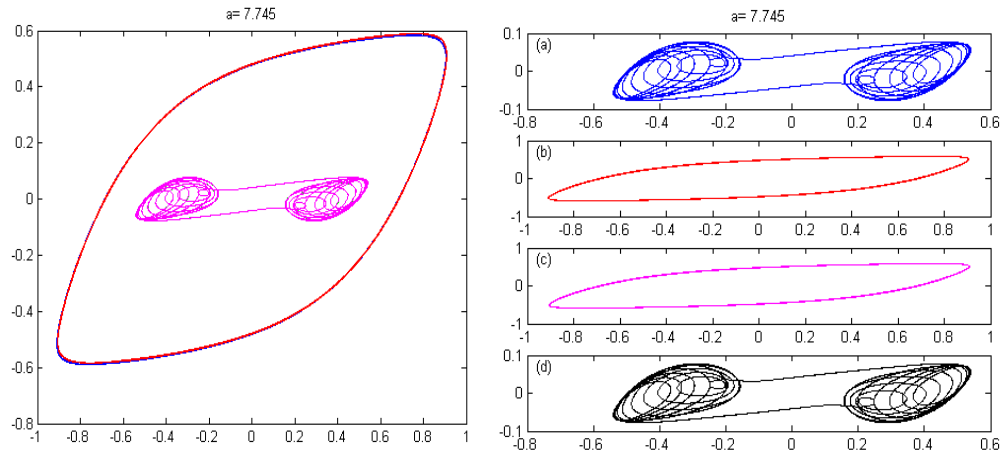


Fig.2 Coexistence of two periodical attractors on the platform of  $(x, y)$  of system (1) when  $a=7.745$ , the right part is the cases for different initial values of system (1), (a) (0.01,0.02,0.03,0.04), (b) (-0.6,-0.5,-0.9,-0.8), (c) (0.6,0.5,0.9,0.8), (d) (-0.01,-0.02,-0.03,-0.04) .

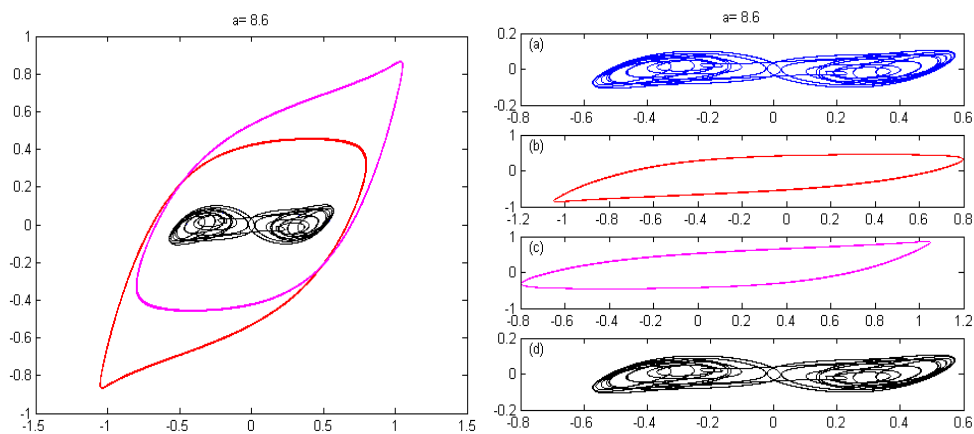


Fig.3 Coexistence of three periodical attractors on the platform of  $(x, y)$  of system (1) when  $a=8.6$ , the right part is the cases for different initial values of system (1), (a) (0.01,0.02,0.03,0.04), (b) (-0.6,-0.5,-0.9,-0.8), (c) (0.6,0.5,0.9,0.8), (d) (-0.01,-0.02,-0.03,-0.04) .