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# Wavelet Preconditioners of Electrohydrodynamic flow problem

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**Abstract.** In this paper wavelet preconditioned method is used for the solution of Electrohydrodynamic flow problem. A finite difference method is used for the solution of Electrohydrodynamic flow equation. The method comprise the nonlinear Newton iteration on the outer loop and a linear iteration on the inside loop where wavelet based preconditioned GMRES (Generalized Minimum Residual) method is used. In the scheme the Jacobian vector product is approximated accurately with much ease (without forming Jacobian explicitly and requiring no extra storage). It overcomes the limitations of conventional schemes for the numerical solution of Electrohydrodynamic flow problem, for computing fluid velocity, covering wide range of Hartmann electric number (*Ha*) parameter with constant $\alpha$ of practical interest. To confirm and validate the solutions obtained, by the present method, are compared with those obtained by GMRES method.

**Keywords:** Classical preconditioners, Wavelet preconditioners, Electrohydrodynamic flow, Hartmann electric number.

## 1. Introduction

Wavelet analysis is a new numerical tool that allows one to represent a function as a linear combination of building blocks (a basis), called wavelets, which are localized in both translation and dilation. Good wavelet localization properties in physical and wave number spaces are to be contrasted with the spectral approach, which employs infinitely differentiable functions but with global support and small discrete changes in the resolution. The various types of wavelets have been used in current research areas in which haar wavelet is the simplest wavelet because of simple applicability, orthogonality and compact support. The haar wavelet based techniques has been successfully used in various applications such as time–frequency analysis, signal de-noising, numerical approximation and solving differential equations (Chen and Hsiao [1], Hsiao and Wang [2], Hsiao [3] and Lepik [4-6]).

Most of the technical problems and engineering phenomena are frequently defined based on the nonlinear differential equations. It is essential to remember that except a limited number of these problems, finding efficient solution is difficult. Therefore, researchers used some classical numerical methods. The classical numerical methods such as the Newtons method (NM), ILU, GMRES are useful numerical techniques ones, which present the numerical solutions for nonlinear differential equations. In this paper, numerical solution of electrohydrodynamic flow problem of a fluid in an ion drag configuration in a circular cylindrical conduit is presented using wavelet preconditioners. The electrohydrodynamic flow of a fluid and its governing equations is considered [7]. The electrohydrodynamic flow is important in analysis of the flow meters, accelerators, pumps and magnetohydrodynamic generators. The differential equation of the problem is the nonlinear singular boundary value problem.

$$y''+\frac{1}{x}y'+Ha\left(1-\frac{y}{1-\alpha y}\right)=0,\quad 0<x<1 \tag{1.1}$$

$$y'(0)=0,\quad y(1)=0 \tag{1.2}$$

where $y$ is the velocity of the fluid, $x$ is the radial distance from the center of cylindrical conduit, $Ha$ is a constant (Hartmann electric number) and $\alpha$ is also a constant that shows the nonlinearity of the problem. Existence and uniqueness of solution of Eqs. (1.1) and (1.2) are discussed by Paullet [8].

As we know, most of the flow and heat transfer equations are nonlinear and usually have not an efficient solution. So, numerical techniques are used many researchers to solve such equations. The most known numerical methods used to solve electrohydrodynamic flow problem is the Newtons, ILU and

GMRES etc. these classical methods gives slow convergence as well as more computational time. To overcome these limitations, a significant challenge is to solve the problem efficiently with faster convergence and less computational time, because Eq. (1.1) is a singular nonlinear BVP, and the type of nonlinearity is in the form of a rational function. Numerical solution of this problem is presented using wavelet based preconditioners.

The present work is organized as follows; Preliminaries are given in section 2. Method of solution with numerical experiment is discussed in section 3. Finally, conclusion of the proposed work is drawn in section 4.

## 2. Preliminaries

Wavelets are functions generated from one single function called the mother wavelet by the simple operations of dilation and translation. A mother wavelet gives rise to a decomposition of the Hilbert space $L^2(\mathrm{R})$, into a direct sum of closed subspaces $W_j,\ j \in \mathrm{Z}$. [9]

Let

$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$$

and

$$W_j = clos_{L^2(\mathrm{R})}\left[\psi_{j,k} : k \in \mathrm{Z}\right] \tag{2.1}$$

Then every $f \in L^2(\mathrm{R})$ has a unique decomposition

$$f(x) = \ldots + s_{-1} + s_0 + s_1 + \ldots \tag{2.2}$$

Where $s_j \in W_j$ for all $j \in \mathrm{Z}$, it is

$$L^2(\mathrm{R}) = \sum_{j \in \mathrm{Z}} W_j = \ldots \oplus W_{-1} \oplus W_0 \oplus W_1 \oplus \ldots \tag{2.3}$$

Using this decomposition of $L^2(\mathrm{R})$, a nested sequence of closed subspaces $V_j,\ j \in \mathrm{Z}$ of $L^2(\mathrm{R})$ can be obtained, defined by

$$V_j = \ldots \oplus W_{j-2} \oplus W_{j-1} \tag{2.4}$$

These closed subspaces $\{V_j,\ j \in \mathrm{Z}\}$ of $L^2(\mathrm{R})$, form a "multiresolution analysis" with the following properties:

i) $\ldots \subset V_{-1} \subset V_0 \subset V_1 \subset \ldots$

ii) $\mathrm{clos}_{L^2}\left(\bigcup V_j\right) = L^2(\mathrm{R})$

iii) $\bigcap V_j = \{0\}$

iv) $V_{j+1} = V_j \oplus W_j$

v) $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1},\ j \in \mathrm{Z}$

Let $\phi \in V_0$ the so-called "scaling function" that generates the multiresolution analysis $\{V_j\}_{j \in \mathrm{Z}}$ of $L^2(\mathrm{R})$. Then

$$\{\phi(-k) : k \in \mathrm{Z}\} \tag{2.5}$$

is a basis of $V_0$, and by setting

$$\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k) \tag{2.6}$$

it follows that, for each $j \in \mathrm{Z}$, the family

$$\{\phi_{j,k} : k \in \mathrm{Z}\} \tag{2.7}$$

is also a basis of $V_j$.

Then, since $\phi \in V_0$ is in $V_1$ and since $\{\phi_{1,k} : k \in \mathrm{Z}\}$ is a basis of $V_1$, there exists a unique sequence $a_k$ that describes the following "two-scale relation":

$$\phi(x) = \sum_{k=-\infty}^{\infty} a_k\, \phi(2x - k) \tag{2.8}$$