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Adaptive synchronization of fractional-order Lorenz systems with memristor

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Abstract. In this paper, adaptive synchronization of fractional-order Lorenz systems with memristor is investigated. Based on the Lyapunov stability theories, several novel criteria are adopted to realize the synchronization of fractional-order Lorenz systems with memristor. Finally, some numerical examples are exploited to demonstrate the theoretical results.

Keywords: Fractional-order system; Memristor; Synchronization; Lorenz system.

1. Introduction

In 1971, the existence of memristor, as the fourth fundamental circuit element included along with the resistor, capacitor and inductor, was theoretically postulated by Chua [1] and experimentally confirmed by the researchers of Hewlett-Packard (HP) who reported on the first memristor device in 2008 [2,3]. The main advantage of the memristor is that the value of resistance would depend on the polarity e and magnitude of the voltage, and also remember the current resistance when the voltage is turned off. On the other hand, fractional calculus, which mainly deals with derivatives and integrals of arbitrary order, was firstly introduced 300 years ago. However, it is only in recent decades that fractional calculus is applied to physics and engineering [4,5]. The major merit of fractional calculus, different from integer calculus, lies in the fact that it has memory and has proven to be a very suitable tool for describing memory and hereditary properties of various materials and processes, it has been widely used in many research fields such as fluid mechanics [6], physics [7] and control processing [8]. Meanwhile, the chaos synchronization is applied in many fields such as cryptography [9], electromagnetic field [10,11], secure communication [12] and bioengineering [13]. Therefore, many scholars investigated the chaos synchronization problems [14-23].

Based on the above analysis of motivation, this paper will study the adaptive synchronization strategy of fractional-order Lorenz systems with memristor, it has a great value in practical applications. Finally, some examples are exploited to demonstrate the theoretical results via numerical simulations.

The rest of this paper is organized as follows. Section 2 describes the model formulation and some basic definitions. In Sect. 3, the adaptive synchronization of fractional-order Lorenz systems with memristor is realized. The numerical examples are provided to show the effectiveness of our theoretical results in Sect. 4. In Sect. 5, some conclusions are proposed.

2. Preliminaries

In this section, some basics of fractional calculus, definitions and lemmas assumptions are recalled. In addition, we introduce the mathematic model of fractional-order chaotic systems with memristor. For convience, $\mathbb{R}^n$will be the n-dimensional Euclidean space with norm $\|\cdot\|$in the all following.

2.1. Definitions and Lemmas

Definition 2.1.1: [4]. Caputo’s fractional derivative of order q for a function $f(t): [0,+\infty) \to R$ is defined by

$$ {}_0^cD_t^q f(t) = \frac{1}{\Gamma(n-q)} \int_0^t \frac{f^n(\tau)}{(t-\tau)^{q-n+1}} d\tau , $$

where $t \geq 0$, n is a positive integer such that $n-1<q<n$, and $\Gamma(\cdot)$ is the gamma function, that is $\Gamma(\tau)=\int_0^\infty t^{\tau-1}e^{-t}dt$ and $\Gamma(\tau+1)=\tau\Gamma(\tau)$.

Moreover, when $0<q<1$,

$$ {}_0^cD_t^q f(t)=\frac{1}{\Gamma(1-q)}\int_0^t \frac{f'(\tau)}{(t-\tau)^q}d\tau . $$

For simplicity, we denote $D^q f(t)$ as the ${}_0^cD_t^q f(t)$, and describe all of the following Caputo operators.

Definition 2.1.2: [4]. Mittag-Leffler function is defined by

$$ E_q(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(kq+1)}, $$

where $q>0$ and $z \in C$.

Lemma 2.1.1: [24] For $q\in(0,1)$, suppose $x(t)\in R^n$ is continuous function, the following inequality holds

$$ \frac{1}{2}D^q x^T(t)x(t) \leq x^T(t)D^q x(t). $$

2.2. Assumptions

Assumption 2.2.1: It is assumed that there is positive constant η such that for any $x, y \in R^n$,

$$ f(y)-f(x) \leq \eta(y-x). $$

Assumption 2.2.2: It is assumed that system (5) is a chaotic system with bounders, that is to say, there are positive constants M_1, M_2 such that

$$ |x_1(t)| \leq M_1, \qquad |x_4(t)| \leq M_2 $$

2.3. Model description

In [25], ϕ is flux, $W(\phi)$ is the memductance, the flux-controlled memristor is considered by

$$ \begin{cases} w(\phi)=a\phi+b\phi^3, \\ W(\phi)=\dfrac{dw(\phi)}{d\phi}=a+3b\phi^2, \end{cases} \tag{2.3.1} $$

Similar to [25], a modified Lorenz system with a memristor can be described by

$$ \begin{cases} \dot{x}_1=-\alpha x_1+\beta x_2-W(x_4)x_1, \\ \dot{x}_2=\gamma x_1-x_2-x_1x_3, \\ \dot{x}_3=x_1x_2-\xi x_3, \\ \dot{x}_4=-x_1. \end{cases} \tag{2.3.2} $$

Refer to the above model, the new fractional-order Lorenz system with memristor according to (2) is described as

$$ \begin{cases} D^q x_1=-\alpha x_1+\beta x_2-W(x_4)x_1, \\ D^q x_2=\gamma x_1-x_2-x_1x_3, \\ D^q x_3=x_1x_2-\xi x_3, \\ D^q x_4=-x_1. \end{cases} \tag{2.3.3} $$

Usually, in order to obtain the chaos generation, we set $q=0.97$, $a=0.1\times10^{-3}$, $b=0.1\times10^{-3}$, $\alpha=8$, $\beta=12$, $\gamma=30$, $\xi=2$, than the simulation is done with the initial value (1, 2, -3, -5) to system (3), the simulation results are shown in Fig. 2.3.1.