ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 12, No. 4, 2017, *pp*.264-269

Image encryption based on fractional order 4-D system with memristor

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(Received March 07, 2017, accepted September 16, 2017)

Abstract. By combining fractional differential operator with the memristor-based 4-D system, a fractional order 4-D system with memristor is proposed and the chaotic behavior of it is investigated. According to the chaos characteristics of the proposed system, an image encryption method is put forward. Numerical simulations suggest that the given image encryption method can make an image effectively resist statistics attack in transferring and saving.

Keywords: Image encryption, fractional order, memristor.

1. Introduction

As a classical mathematical notion, fractional calculus is a generalization of the integer-order differentiation. When it was first proposed, due to its complexity and lack of application background, fractional order differential equation did not attract much attention of researchers. Until recent decades, it is proved that fractional order differential equations play an important role and can better describe many systems in science and engineering [1-4].

In recent years, some researchers made much progress in the mathematical construction and theoretical analysis of fractional chaotic systems. It was found that some chaotic systems still present chaos or hyperchaos when the order of these systems is a fraction, such as Lorenz system, Chua's system, Rössler system, Chen system, Lü system. Fractional order Chua's system was studied and a result was obtained that the system could produce chaos when the order of it is less 3 [5, 6]. The least order for fractional order Rössler to present chaos is 2.4 and the least order for fractional order Rössler to appear hyperchaos is 3.8 [7]. The fractional order Chen system is analyzed using fractional calculus theory [8-10]. With the development of chaos or hyperchaos, it has been applied into much fields, such as physics, biology, electrical engineering, communication theory, etc. [11-16].

Based on above, in this paper, the dynamics of a fractional order 4-D system with memristor is investigated and the condition to generate chaot is found. According to this feature, the proposed fractional order 4-D system with memristor is applied into image encryption.

2. Preliminary and system description

In this section, definition of Caputo's fractional derivative of order for a function is introduced to discuss the dynamics of fractional order 4-D system with memristor and the fractional order 4-D system to be investigated is depicted.

Definition 2.1: [1]. Caputo's fractional derivative of order q for function $f \in C^n([0,+\infty), R)$ is defined by

$$ {}_0^cD_t^q f(t) = \frac{1}{\Gamma(n-q)} \int_0^t \frac{f^n(\tau)}{(t-\tau)^{q-n+1}} d\tau , \tag{2.1} $$

where $t \ge 0$, n is a positive integer such that $n-1 < q < n$. $\Gamma(\cdot)$ is gamma function satisfying $\Gamma(\tau) = \int_0^\infty t^{\tau-1} e^{-t} dt$ and $\Gamma(\tau+1) = \tau\Gamma(\tau)$.

From formula (1), we can know that, for $0 < q < 1$, it can be obtained

$$ {}_0^cD_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_0^t \frac{f'(\tau)}{(t-\tau)^q} d\tau . $$

It should be noted that, for simplicity, we denote ${}_0^cD_t^q f(t)$ as $D^q f(t)$ in following operator.

According to the method reported in [17], a new memristive system had been constructed by utilizing memristor to substitute resistor in a three-dimensional hypogenetic chaotic jerk system proposed by [18], which is given as

$$ \begin{cases} \dot{x} = |y| - b \\ \dot{y} = W(w)z \\ \dot{z} = |x| - y - az - c \\ \dot{w} = z \end{cases}, \tag{2.2} $$

where x, y, z are state variables, w is the inner dimensionless state variable of the proposed memristor, and $W(w) = \alpha - \beta|w|$ is the memductance function used to describe an ideal and active flux-controlled memristor with an absolute value nonlinearity. The dynamical behaviors of memristive system (2) have been studied and initial condition-dependent dynamical phenomenon of extreme multistability has been found [19].

Learned from model (2), the corresponding fractional order system with memristor is described as

$$ \begin{cases} D^q x = |y| - b \\ D^q y = W(w)z \\ D^q z = |x| - y - az - c \\ D^q w = z \end{cases}. \tag{2.3} $$

When the parameters are selected as a = 0.6, b = 1.3, c = 2, α= 1, β= 0.1 and the initial conditions are set as (0, 0, 0.001, 0), the lookalike chaotic attractor can be observed in the fractional order system (3) at q=0.98, which can be seen in Fig. 1. Furthermore, time series of variables x and z are given in Fig.2 and Fig.3, respectively, which verifies that the variables can appear chaotic behavior for appropriate parameters.

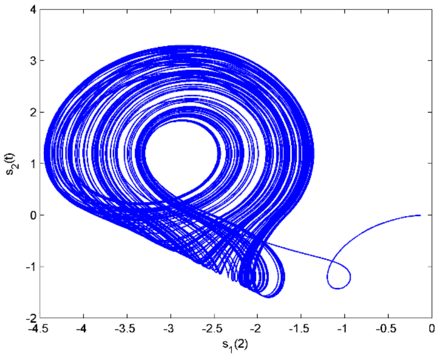

Fig-2.1:- Lookalike chaotic attractor of system (3) for q=0.98.

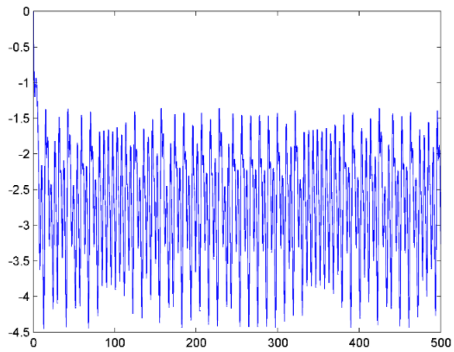

Fig-2.2:- Time series of variable x in system (3).