ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 12, No. 4, 2017, *pp*.291-303

Converting Z-number to Fuzzy Number using Fuzzy Expected Value

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(Received March 20, 2017, accepted May 11, 2017)

Abstract. In order to deal with uncertain information of real-world, in 2011 Zadeh suggested the concept of a Z-number, as an ordered pair of fuzzy numbers ($\tilde{A}$,$\tilde{B}$) that describes the restriction and the reliability of the evaluation. Due to the limitation of its basic properties, converting Z-number to classical fuzzy number is rather significant for application. In this paper, we will calculate fuzzy expected value of a Z-number with assuming uniform distribution and linear membership functions. This fuzzy expected value can be used instead of Z-number in applications. An Example is used to illustrate the procedure of the proposed approach.

Keywords: Z-number; Fuzzy Expected value; Fuzzy Set

1. Introduction

In the real world, decisions are based on information usually uncertain, imprecise and/or incomplete, such information is often characterized by fuzziness, but it is not sufficient and the other important property of information is that information must be reliable. Thus, fuzziness from the one side and reliability form the other side are strongly associated to each other. In order to take into account this fact, in 2011 Zadeh proposed a concept, namely Z-number, which is an order pair of fuzzy numbers ($\tilde{A}$, $\tilde{B}$). The first component, $\tilde{A}$, is a fuzzy restriction and the second component $\tilde{B}$ is a reliability of the first component. Typically, $\tilde{A}$ and $\tilde{B}$ are described in a natural language for example (about 25 minutes, very sure) [1, 2, 3].

In 2012, Yager used Z-number to provide a simple illustration of a Z-valuation (X, $\tilde{A}$, $\tilde{B}$). He showed that these Z-valuations essentially induce a possibility distribution G(p) over probability distributions associated with an uncertain variable X and used this representation to make decisions and answer questions. He suggested manipulation and combination of multiple Z-valuations. He showed the relationship between Z-numbers and linguistic summaries and provided for a representation of Z-valuations in terms of Dempster–Shafer belief structures, which made use of type-2 fuzzy sets [4,5].Yager found a fuzzy expected value of a Z-number whose probability density function is expressible in terms of an exponential distribution.

Kang et al. [6] proposed a simple method for converting Z-numbers with range [0, 1] to the classical fuzzy numbers in 3 step: first convert the second part (reliability) into a crisp number and then calculate the weighted Z-number by adding the weight of the second part (reliability) to the first part (restriction) and finally converting the weighted fuzzy number to normal fuzzy number. The benefit of the proposed method is represented by its low analytical and computational complexity, hence their theorem for converting Z-number to classical fuzzy number was used in some papers [7,8,9,10].

Gardashova [11] suggested an algorithm of decision making method using Z-numbers, in 5 steps: Construction of the fuzzy decision making matrix, transforming the linguistic value to numerical value, normalizing the fuzzy decision making matrix, Converting the Z-numbers to crisp number and Determining the priority weight of each alternative. He used a simple way of the canonical representation of multiplication operation on triangular fuzzy numbers [12] for the converting the Z-numbers to crisp number.

In this study we find a fuzzy expected value of a Z-number whose probability density function is expressible in terms of a uniform distribution. The result can be used in many applications to converting Z-number to fuzzy number.

The remainder of the paper is organized as follows: In Section 2 we discuss the concept of Z-number, Z-number with the probability based on uniform distribution and the probability of a restriction ($\tilde{A}$) on the values of uncertain variable (X) based on the parameters of uniform distribution. Section 3 explains the method of calculating the maximum probability of $\tilde{A}$. Later in section 4, the membership function of the possibility of the expected value of Z-number (μ) is proposed. Finally, the whole presented material is summed up in section 5.

2. The concept of Z-numbers

Any Z-number is an ordered pair of fuzzy numbers $(\tilde{A},\tilde{B})$ where $\tilde{A}$ is a restriction on the values of uncer tain variable, X, and $\tilde{B}$ is the level of confidence in $\tilde{A}$. Both $\tilde{A}$ and $\tilde{B}$ are fuzzy sets. Zadeh[1] defines Z-valuat ion as an ordered triple, (X,$\tilde{A}$,$\tilde{B}$), which is equivalent to " X is $(\tilde{A},\tilde{B})$". Moreover, Z-valuation can be defined as

$$Prob(X\ \ is\ A^{\sim})\ is\ B^{\sim} \tag{1}$$

According to the Zadeh's equation, we express the probability that X is $\tilde{A}$ as:

$$Prob_p\big(X\ is\ \tilde{A}\big) = \int_R\ A(x)f(x)dx \tag{2}$$

where *A(x)* is the membership function of Ã and f(x) is the probability density function of X on R. Now, we can get *G(p)*, the degree to which p satisfies our Z-valuation, 〖Prob〗_p (X is A˜) is B, as:

$$G(p) = B\left(Prob_p(X\ is\ A)\right) = B(\int_R\ A(x)f(x)dx) \tag{3}$$

Since the probability density function of X is unknown, according to Zadeh's simplifying assumption, a particular set of parametric distributions (e.g. normal, exponential and uniform distributions) can be applied based on available knowledge about the variable [1]. But when the knowledge available is not sufficient, the uniform distribution is proposed for a poorly known variable [13]. This distribution is sometimes referred to as the "no knowledge" distribution. Hence, we used uniform distribution in the current study.

Let *(A˜,B˜)* be a Z-number,where Ã is a triangular number, (*c-l* , *c* , *c+r*), with a membership function as follows:

$$A(x) = \begin{cases} 0 & if \quad x < c-l \\ (x-c+l)/l & if \quad c-l \le x < c \\ (c+r-x)/r & if \quad c \le x \le c+r \\ 0 & if \quad x > c+r \end{cases} \tag{4}$$

On the other hand, B˜, confidence, could be from the set {"Likely", "Usually", "Sure"} are modelled using the right hand fuzzy sets as follows (see Fig. 1)

$$B_L(P) = \begin{cases} 0 & P < 0.5 \\ \frac{P-0.5}{0.1} & 0.5 \le P \le 0.6 \\ 1 & 0.6 < P \end{cases} \tag{5}$$

$$B_U(P) = \begin{cases} 0 & P < 0.65 \\ \frac{P-0.65}{0.1} & 0.65 \le P \le 0.75 \\ 1 & 0.75 < P \end{cases} \tag{6}$$

$$B_S(P) = \begin{cases} 0 & P < 0.8 \\ \frac{P-0.8}{0.1} & 0.8 \le P \le 0.9 \\ 1 & 0.9 < P \end{cases} \tag{7}$$

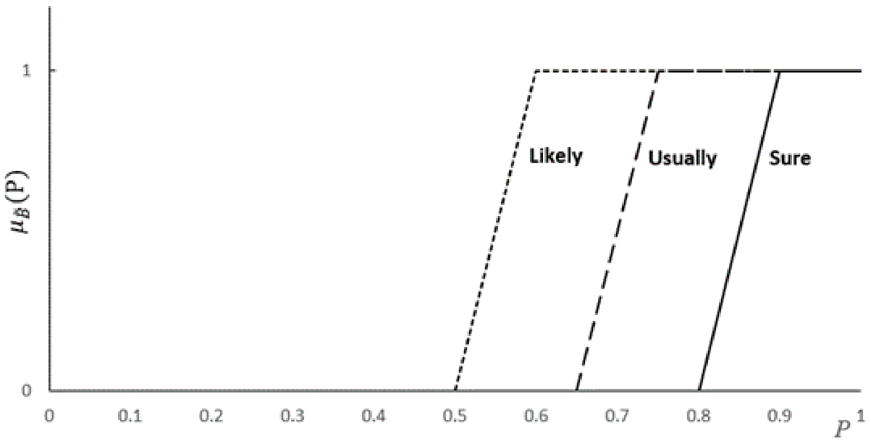

Fig 1. Fuzzy sets of linguistic reliability values.

Assuming that the probability density function, f(x), is uniform as:

$$f(x) = \frac{1}{b-a}, a \le x \le b. \tag{8}$$