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# Finite-time chaos synchronization of the delay Lorenz system with disturbance via a single controller

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**Abstract.** This paper deals with the finite-time chaos synchronization of the delay Lorenz system with disturbance via a single controller. Based on the finite-time stability theory, a control law is proposed to realize finite-time chaos synchronization of delay Lorenz system with disturbance. Finally, numerical simulation results are given to demonstrate the effectiveness and robustness of the proposed scheme.

**Keywords:** Finite-time chaos synchronization; delay Lorenz system; disturbance

## 1. Introduction

Chaos synchronization has attracted much attention of many researches since the seminal work of Pecora and Carroll [1]. From then on, chaos synchronization has been developed extensively and intensively due to its potential application in many fields, such as secure communication [2, 3], complex networks [4-7], biotic science [8-13] and so on [14-26].

Nowadays, most of the results about chaos control and synchronization are derived based on the asymptotic stability of the chaotic systems. In fact, it is more valuable to control or synchronize chaotic systems as fast as possible. To obtain faster convergence, the finite-time control approach is an effective technique. Moreover, the finite-time techniques have demonstrated better robustness and disturbance rejection properties than that of asymptotic methods [27-37]. Therefore, the finite-time chaos control and synchronization have attracted a great deal of attention over the last few decades.

On the other hand, it is difficult to know the external disturbance always occurs in the system. Thus, the chaos control and synchronization of chaotic system in the presence of external disturbance are effectively crucial in practical applications.

In this paper, we present a controller to realize finite-time synchronization of delay Lorenz system with disturbance. The controller is robust and simple to be constructed. Numerical simulations are presented to demonstrate the effectiveness and robustness of the proposed scheme.

## 2. Preliminary definitions and lemmas

Finite-time synchronization means that the state of the slave system can track the state of the master system after in finite-time. The precise definition of finite-time synchronization is given as below.

**Definition 1.** Consider the following two chaotic systems:

$$\dot{x}_m = f(x_m),$$
$$\dot{x}_s = h(x_m, x_s), \tag{1}$$

where $x_m, x_s$ are two $n$-dimensional state vectors. The subscripts '*m*' and '*s*' stand for the master and slave systems, respectively. $f: R^n \to R^n$ and $h: R^n \to R^n$ are vector-valued functions. If there exists a constant $T > 0$, such that

$$\lim_{t\to T}\|x_m - x_s\| = 0,$$

and

$$\|x_m - x_s\| \equiv 0, \quad \text{if } t \geq T,$$

then synchronization of the system (1) is achieved in a finite-time.

**Lemma 1** [32]**.** Assume that a continuous, positive-definite function $V(t)$ satisfies differential inequality

$$\dot{V}(t) \le -cV^{\eta}(t),\ \forall t \ge t_0, V(t_0) \ge 0, \tag{2}$$

where $c > 0$, $0 < \eta < 1$ are constants, then, for any given $t_0$, $V(t)$ satisfies inequality

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ t_0 \le t \le t_1, \tag{3}$$

and

$$V(t) \equiv 0,\ \forall t \ge t_1, \tag{4}$$

with $t_1$ given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

**Proof.** Consider differential equation

$$\dot{X}(t) = -cX^{\eta}(t),\ X(t_0) = V(t_0), \tag{6}$$

although differential equation (6) does not satisfy the global Lipschitz condition, the unique solution of Eq.(6) can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0). \tag{7}$$

Therefore, from the comparison Lemma, one obtains

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ t_0 \le t \le t_1, \tag{8}$$

and

$$V(t) \equiv 0,\ \forall t \ge t_1.$$

with $t_1$ given in (5).

**Lemma 2** [34]. If $\alpha > (\frac{2}{3})^{\frac{2}{3}}$, it can be gotten that

$$(\alpha|x_1| + \frac{1}{2}x_2^{\,2})^{\frac{3}{2}} + x_1x_2 \ge 0, \tag{9}$$

where $x_1$ and $x_2$ are any real numbers .

**Corollary 1** [34]. If $\alpha > (\frac{2}{3})^{\frac{2}{3}}$, it can be obtained that

$$|x_1x_2| \le (\alpha|x_1| + \frac{1}{2}x_2^{\,2})^{\frac{3}{2}}, \tag{10}$$

where $x_1$ and $x_2$ are any real numbers .

## 3. Main results

A chaotic system is extremely sensitive to disturbance. In actual situation, the system is disturbed and cannot be exactly predicted. These uncertainties will destroy the synchronization and even break it. Therefore, it is important and necessary to study the synchronization of systems with disturbance. In this section, the dynamic behaviors of the delay Lorenz system is to be explored and the finite-time synchronization of the delay Lorenz systems will be discussed.

**3.1 Dynamics of delay Lorenz system with disturbance**

Delay Lorenz system with disturbance is considered as

$$\dot{x}_1 = a(x_2 - x_1),$$
$$\dot{x}_2 = cx_1 - x_2 - x_1x_3 + Ax_2\sin(\omega t),$$
$$\dot{x}_3 = x_1x_2 - bx_3(t-\tau), \tag{11}$$