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Wavelet based numerical solution of linear and non-linear parabolic partial differential equations using Lifting scheme

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Abstract. Partial differential equations are fundamental in modeling several natural phenomena. The present work is designed for the Wavelet based numerical solution of linear and non-linear parabolic partial differential equations using lifting scheme. To demonstrate the efficiency and competence of the proposed scheme, we used both orthogonal and biorthogonal wavelets. This scheme speeds up convergence in lesser computational time as compared with existing schemes. Some test problems are presented for the validity and applicability of the scheme.

Keywords: Parabolic partial differential equations: Lifting scheme; Orthogonal and Biorthogonal wavelets.

1. Introduction

In recent years there has been much attention for finding the numerical solution of partial differential equations (PDEs). In general parabolic PDEs with or without reaction terms are used as a fundamental tool to model a wide class of phenomena occurring in physical and biological sciences, such as Heat equation, Wave equation, Fisher's equation, Cahn-Allen equation, Burgers equation and many other important equations. These equations are usually difficult to solve analytically, so these are required to obtain efficient approximate solutions. The importance of obtaining the exact or approximate solutions of nonlinear partial differential equations in physics and mathematics is still a significant problem that needs new methods to discover exact or approximate solutions. Due to this, there is a demand on the development of accurate and efficient analytical or numerical methods able to deal with these equations.

Recently, some of the iterative methods are used for the numerical and analytical solutions of Linear and Nonlinear partial differential equations. For example, He's variational iteration technique [1], the homotopy perturbation method [2], Adomian decomposition method [3] etc. Mathematical models of basic flow equations which describe unsteady transport problems are governed by a single or a system of nonlinear PDEs.

Analytical solution of certain parabolic PDEs either does not exist or is hard to find. Due to this fact, in the last decades, there have been great advances in the development of finite difference, finite element, spectral techniques and finite volume methods for the solution of parabolic PDEs. The parabolic PDEs of the forms [4],

$$u_t = u_{xx} + f(u), \qquad 0 \le x \le 1,\ t > 0 \tag{1.1}$$

$$u_t = u_{xx} + g(x,t), \qquad 0 \le x \le 1,\ t > 0 \tag{1.2}$$

subject to initial condition (IC) and boundary conditions (BCs). Where and are the functions of dependent and independent variables.

The purpose of this paper is to give a numerical solution for a class of linear and non-linear parabolic partial differential equations by Lifting scheme using different wavelets.

Wavelet theory had been developed independently on several fronts. Different signal processing techniques, developed for signal and image processing applications, had significant contribution in this development. Some of the major contributors to this theory are: multiresolution signal processing used in computer vision; sub band coding, developed for speech and image compression; and wavelet series expansion, developed in applied mathematics. Using different wavelets, various numerical methods have been applied for solving PDEs from beginning of the early 1990s. In the last two decades this problem has attracted great attention and numerous papers about these topics have been published. Wavelets permit the perfect representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithm [5]. Wavelet based numerical methods are used for solving the system of equations with faster convergence and lower computational cost.

Some of the earlier works on wavelet based methods can be found in [6]. A collection of the discrete wavelet transforms (DWT) and the FAS were introduced recently in [7-9]. The wavelet based full approximation scheme (WFAS) has exposed to be a very efficient and favorable method for numerous problems related to computational science and engineering fields [10]. These methods can be either used as an iterative solver or as a preconditioning technique, offering in many cases a better performance than some of the most innovative and existing FAS algorithms. Due to the efficiency and potentiality of WFAS, researches further have been carried out for its enrichment. In order to realize this task, work build that is orthogonal/biorthogonal discrete wavelet transform using lifting scheme [11]. Wavelet based lifting technique is introduced by Sweldens [12], which permits some improvements on the properties of existing wavelet transforms. The technique has some numerical benefits as a reduced number of operations which are fundamental in the context of the iterative solvers. Evidently all attempts to simplify the wavelet solutions for PDE are welcome. In PDE, matrices arising from system are dense with non-smooth diagonal and smooth away from the diagonal. This smoothness of the matrix transforms into smallness using wavelet transform and it leads to design the effective wavelets based lifting scheme.

Lifting scheme is a new approach to construct the so-called second generation wavelets that are not necessarily translations and dilations of one function. The latter we refer to as a first generation wavelets or classical methods. The lifting scheme has some additional advantages in comparison with the classical wavelets. This transform works for signals of an arbitrary size with correct treatment of boundaries. Another feature of the lifting scheme is that all constructions are derived in the spatial domain. This is in contrast to the traditional approach, which relies heavily on the frequency domain. Staying in the spatial domain leads to two major advantageous: i) It does not require the machinery of Fourier analysis as a prerequisite, this leads to a more intuitively appealing treatment better suited to those in interested in applications than mathematical foundations. ii) The algorithms that can easily be generalized to complex geometric situations, this leads to second generation wavelets. In addition, the lifting scheme makes a computational time optimal and sometimes increasing the speed of calculations.

The lifting scheme starts with a set of well-known filters, thereafter lifting steps are used an attempt to improve (lift) the properties of a corresponding wavelet decomposition. This procedure has some mathematical benefits as a reduced number of operations which are essential in the context of the iterative solvers. In addition to this, the present paper illustrates that the application of the lifting technique to the real world problems.

The present paper is organized as follows: Section 2 deals with Preliminaries of wavelet filter coefficients and Lifting scheme. Method of solution is discussed in Section 3. Section 4 provides Numerical results of the test problems and finally, in section 5 conclusions of the proposed work are discussed

2. Related Work

The important feature of the lifting scheme is that every filter bank based on lifting automatically satisfies perfect reconstruction properties. The lifting scheme starts with a set of well-known filters; thereafter lifting steps are used in attempt to improve the properties of corresponding wavelet decomposition.

Now, we have discussed about different wavelet filters as follows:

a) Haar wavelet filter coefficients

We know that low pass filter coefficients $\left[a_0,\ a_1\right]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ and high pass filter coefficients $\left[b_0,\ b_1\right]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ play an important role in decomposition. Thus it is natural to wonder that it possible to model the decomposition in terms of linear transformations i.e. matrices. Moreover, since digital signals and images are composed of discrete data, we need a discrete analog of the decomposition algorithm so that we can process signal and image data.