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Research on the rumor spreading model with noise inference in the homogeneous network

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Abstract. In this paper, a dynamic I2SR model based on rumor spreading with random noise interference is proposed. Active state of the spreaders and the random noise interference in the homogeneous network can be considered. This paper divided spreaders into high probability spreaders and low probability spreaders that can be transformed. The existence and uniqueness of global positive solutions, extinction , and the spreading range of rumors be proved. Through numerical simulation, we have three conclusions as follows: (1)The spreaders can quickly reach to the peak in the shorter time with the bigger of the transmissions rate;(2)The high probability spreaders not only pay more attention to rumors but also more likely to spread;(3)There have a negative correlation between the spreading range of rumors and noise intensity.

Keywords: rumor spreading，active state，noise interference，complex network

1. Introduction

Rumors are often defined as the spreading of things that are of interest to the public or the interpretation of unconfirmed events through various ways. Rumor as an important method of social topics, its spreading pay more attention to people's lives. Rumor can lead to people reputation damage to some extent[1], the community or the company has an immeasurable consequences[2,3]. Because of its important role in society, many scholars began to study the spreading of rumors[4,5].

Because of the importance of the infectious disease model, it is widely used in various fields, especially the rumor spreading model [6], Delay and Kendall introduced the D-K model based on the classical epidemic disease model in 1965, and the difference between the spreading of diseases and rumors was analyzed. In this model, the total population is divided into three groups: one kind of people who has never heard rumors; one who has heard and spread it; one is a person who knows but never spread it. These groups are called "ignorant", "spreaders", "stiflers". Since then, many scholars have begun to study the relationship between rumor spreading and epidemic disease models. Some scholars[7-9] have studied the methods of disease transmission and the spread of rumor on complex networks based on the theory of mean field theory and infiltration. A number of people [10-13] have studied the rumor of the social network topology .In reality, the spreaders shows different interests and motivations due to the different characteristics of rumors and their connection with the individual. Subsequently, Huo and Wang et al. considered the effect of spreaders with different infection rates on rumor spreading in the homogeneous network [14]. However, epidemic disease models are unavoidably disturbed by environmental noise [15-18]. The stochastic model can be used to predict the future dynamic changes of the system. The stochastic differential equation model has also been widely used in the model of noise interference. Cai et al. describes the influence of environmental fluctuation on the infection rate coefficient about SIRS model, it is described as Gaussian white noise. Because of the similarity between rumor spreading and the spreading of epidemic diseases, it is also disturbed by the noise during the rumor spreading [19].

In this paper, we study the spreading of rumors from different perspectives and carry out a system analysis. In reality, spreaders will show different interests and motivations due to the different characteristics of rumor and their connection with the individual. Some people take the initiative to participate in the rumor spreading, others are showing a lower interest. In this paper, we assume that the average degree of the system in the homogeneous network is disturbed by random noise, and that the spreader's active state in the homogeneous network is considered in the process of the rumor spreading, and the spreaders with high activity is active and spread rumors with greater probability; And low active spreaders will show negative reactions, and with a lower probability to spread rumors, under certain conditions they can be transformed into each other. Based on these two factors, we put forward a dynamic I2SR model based on the rumor spreading that is provided with random noise interference at different infection rates. In this paper, we establish the model of stochastic differential equation system on complex networks, and prove the existence and uniqueness of global positive solutions, extinction and the spreading range of rumors. In the numerical simulation section, we take into account the impact of each parameter on rumor spreading, and the results are analyzed and summarized.

2. Model establishment and theoretical analysis

In this paper, we consider that the noise interference is $\langle k\rangle + \theta\dot{B}(t)$ .where $\langle k\rangle$ is the average degree and $\dot{B}(t)$ is the standard Brownian motion on the complete probability space, θ indicates the noise intensity. We analyze the following I2SR model with random noise interference

$$\begin{cases} \dfrac{dI(t)}{dt} = -\lambda_1 I(t)S_1(t)\langle k\rangle - \lambda_2 I(t)S_2(t)\langle k\rangle - \lambda_1 I(t)S_1(t)\theta\dot{B}(t) - \lambda_2 I(t)S_2(t)\theta\dot{B}(t) \\ \dfrac{dS_1(t)}{dt} = \lambda_1 I(t)S_1(t)\langle k\rangle + \alpha S_2(t) - \delta_1 S_1(t) - \beta S_1(t) + \lambda_1 I(t)S_1(t)\theta\dot{B}(t) \\ \dfrac{dS_2(t)}{dt} = \lambda_2 I(t)S_2(t)\langle k\rangle - \alpha S_2(t) - \delta_2 S_2(t) + \beta S_1(t) + \lambda_2 I(t)S_2(t)\theta\dot{B}(t) \\ \dfrac{dR(t)}{dt} = \delta_1 S_1(t) + \delta_2 S_2(t) \end{cases} \tag{1}$$

where $I(t), S_1(t), S_2(t), R(t)$ represent the density of the ignorants, the low probability spreaders, the high probability of spreaders, and the stiflers at time t respectively. When the ignorant nodes is touch with the spreader nodes, some of the ignorant nodes becomes the low probability spreader nodes with the probability λ_1, and some of the ignorant nodes becomes the high probability spreader nodes with the probability λ_2, we suppose $\lambda_2 > \lambda_1$. When the low probability spreader nodes $S_1(t)$ is contact with the stifler nodes, with the probability δ_1 becomes the stiflers, and when the high probability spreader nodes $S_2(t)$ gets contact with the stifler nodes, with the probability δ_2 becomes the stiflers. And with the shrinking of rumor and the increasing of rumor's heat, we consider the probability of low probability and high probability spreaders can be transform. That is, we use the decay rate α to express high probability spreaders to low probability spreaders, the low probability spreaders is transformed into the high probability spreaders with probability β.

2.1. Existence and Uniqueness of Global Positive Solutions

Theorem 2.2.1 For any given initial value $(I(t), S_1(t), S_2(t), R(t)) \in R_+^4$, Then system (1) has a unique positive solution when $t > 0$, and the solution will stay in R_+^4 with probability 1. where

$R_+^4 = \{(I(t), S_1(t), S_2(t), R(t)) \mid 0 < I(t), S_1(t), S_2(t), R(t) < N, I(t) + S_1(t) + S_2(t) + R(t) = N\}$.

Proof Because the coefficients of the system satisfy the local Lipschitz condition, then for any given initial value $(I(0), S_1(0), S_2(0), R(0)) \in R_+^4$ .System (1) has a unique local solution $(I(t), S_1(t), S_2(t), R(t)) \in R_+^4$ on $t \in [0, \tau_e)$, where τ_e is explosion time.

To prove that this solution is global, we just have to prove that $\tau_e = \infty$ a.s is true. To achieve this objective, we assume that $k_0 > 0$ is large enough that $I(0), S_1(0), S_2(0), R(0)$ falls entirely on interval $[1/k_0, k_0]$, and for each integer $k > k_0$, we define the following stopping time