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A New Coding Theory on Fibonacci n-Step Polynomials

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Abstract. In this paper, we develop a new series of Fibonacci n-step polynomials. Based on these series of polynomials, we introduce a new class of square matrix of order n. Thereby, we define a new coding theory called Fibonacci $\boldsymbol{n}$-step polynomials coding theory. Then we calculate the generalized relations among the code elements for all values of n. It is shown that, for $n = 2$, the correct ability of this method is 93.33% whereas for n $=$ 3, the correct ability of this method is 99.80%. The interesting part of this coding/decoding method is that the correct ability does not depend on x and increases as n increases.

Keywords: Fibonacci numbers, Fibonacci n-step numbers, Fibonacci polynomials, Fibonacci n-step polynomials, Fibonacci n-step polynomials coding, Error correction.

1. Introduction

The Fibonacci numbers F_k $(k = 0, \pm 1, \pm 2, \pm 3, \dots)$ is defined by the second-order linear recurrence relation:

$$F_{k+1} = F_k + F_{k-1} \tag{1}$$

with the initial terms $F_0 = 0, F_1 = 1$.

The Fibonacci polynomials are the extension of the Fibonacci numbers, defined by the recurrence relation

$$F_{k+1}(x) = xF_k(x) + F_{k-1}(x) \tag{2}$$

with the initial terms $F_0(x) = 0, F_1(x) = 1$.

The Fibonacci n-step numbers [15] $F_k^{(n)}$ are the generalizations of the Fibonacci numbers, defined by the recurrence relation

$$F_k^{(n)} = F_{k-1}^{(n)} + F_{k-2}^{(n)} + \dots + F_{k-n}^{(n)},$$

$$F_0^{(n)} = F_1^{(n)} = \cdots = F_{n-2}^{(n)} = 0, F_{n-1}^{(n)} = 1 \tag{3}$$

for $k = 0, \pm 1, \pm 2, \pm 3, \dots$ and $n = 1, 2, 3, \dots$.

The first few sequence of Fibonacci n-step numbers are summarized in the Table 1.

Table 1. Fibonacci n-step Numbers

k	-5	-4	-3	-2	-1	0	1	2	3	4	5	Name
$F_k^{(1)}$	1	1	1	1	1	1	1	1	1	1	1	Degenerate
$F_k^{(2)}$	5	-3	2	-1	1	0	1	1	2	3	5	Fibonacci Numbers
$F_k^{(3)}$	-3	2	0	-1	1	0	0	1	1	2	4	Tribonacci Numbers
$F_k^{(4)}$	2	0	0	-1	1	0	0	0	1	1	2	Tetranacci Numbers
$F_k^{(5)}$	0	0	0	-1	1	0	0	0	0	1	1	Pentanacci Numbers

For $k \geq n-1, r_n = \lim_{k\to\infty} \frac{F_k^{(n)}}{F_{k-1}^{(n)}}$ exists, called n- annaci constant and is the real root ≥ 1 of the equation

$$x^n - x^{n-1} - x^{n-2} - \cdots - x - 1 = 0$$

For even n, there are exactly two real roots, one is > 1 and one is < 1 and for odd n, there is exactly one real root, which is always ≥ 1, the equality sign hold if and only if $n = 1$.

Actually, $r_1 = 1, r_2 = 1.61803$ called Golden mean, $r_3 = 1.83929$ called Tribonacci constant, $r_4 = 1.92756$ called Tetranacci constant, $r_5 = 1.96595$ called Pentanacci constant etc. and $\lim_{n\to\infty} r_n = 2$ [6]. In 2006, A. P. Stakhov introduces a new coding theory on Fibonacci matrices [14]. In 2009, M. Basu and B. Prasad describe the generalized relations among the code elements for Fibonacci coding theory [1] and Coding theory on the m-extension of the Fibonacci p-numbers [2]. In 2010, M. Esmaeili and M. Esmaeili introduce a Fibonacci-polynomial based coding method with error detection and correction [12]. After that, M. Basu and M. Das present a new coding theory on Tribonacci matrices [3], coding theory on Fibonacci n-step numbers [4] and coding theory on constant coefficient Fibonacci n-step numbers [5].

In this paper we define Fibonacci n-step polynomials and Fibonacci n-step polynomial matrix of order n. Thereby, we illustrate a new coding theory called Fibonacci n-step polynomials coding theory along with its properties.

Definition 1.1. The Fibonacci n-step polynomials $F_k^{(n)}(x)$ are defined by the recurrence relation

$$F_k^{(n)}(x) = x^{n-1}F_{k-1}^{(n)}(x) + x^{n-2}F_{k-2}^{(n)}(x) + \dots + F_{k-n}^{(n)}(x), \quad (4)$$

with the initial terms $F_0^{(n)}(x) = F_1^{(n)}(x) = \cdots = F_{n-2}^{(n)}(x) = 0, F_{n-1}^{(n)}(x) = 1$, for $k = 0, \pm1, \pm2, \pm3, \dots$ and $n = 1, 2, 3, \dots$.

The first few sequence of Fibonacci n-step polynomials are summarized in the Table 2.

Table 1. Fibonacci n-step polynomials

k	-5	-4	-3	-2	-1	0	1	2	3	4	5
$F_k^{(1)}(x)$	1	1	1	1	1	1	1	1	1	1	1
$F_k^{(2)}(x)$	x^4+3x^2+1	$-(x^3+2x)$	x^2+1	$-x$	1	0	1	x	x^2+1	(x^3+2x)	x^4+3x^2+1
$F_k^{(3)}(x)$	$-(x^4+2x)$	x^3+1	0	$-x$	1	0	0	1	x^2	(x^4+x)	x^6+2x^3+1
$F_k^{(4)}(x)$	(x^4+1)	0	0	$-x$	1	0	0	0	1	x^3	x^6+x^2
$F_k^{(5)}(x)$	0	0	0	$-x$	1	0	0	0	0	1	x^4

Definition 1.2. n- annaci polynomial ratio is defined by $\frac{F_k^{(n)}(x)}{F_{k-1}^{(n)}(x)}, k \geq n-1$. Taking $k \to \infty$, we have $\lim_{k\to\infty} \frac{F_k^{(n)}(x)}{F_{k-1}^{(n)}(x)}$ exists [13] and $\lim_{k\to\infty} \frac{F_k^{(n)}(x)}{F_{k-1}^{(n)}(x)} = r_n(x)$, say which depends on x.

For example, $r_2(x) = \frac{x+\sqrt{x^2+4}}{2}$,

$$r_3(x) = \frac{2^{\frac{1}{3}}x^2 + (2x^6 + 9x^3 + 27 + \sqrt{-19x^6 + 378x^3 + 729})^{\frac{1}{3}} + (2x^6 + 9x^3 + 27 - \sqrt{-19x^6 + 378x^3 + 729})^{\frac{1}{3}}}{2^{\frac{1}{3}} \times 3}.$$

Definition 1.3. Fibonacci n-step polynomial matrix $M_n(x)$ is a square matrix of order n and is given by

$$M_n(x) = \begin{pmatrix} \boldsymbol{x} & 1 \\ I_{n-1} & \boldsymbol{0} \end{pmatrix} = \begin{pmatrix} x^{n-1} & x^{n-2} & x^{n-3} & \cdots & x & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ . & . & . & \cdots & . & . \\ . & . & . & \cdots & . & . \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix}$$