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Application of shadowing Filter in Weather Predictability

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Abstract: The physical meaning of the Ikeda model is analyzed. The influence of the parameters in the model on the state of the solution is discussed. The change process of the solution when the parameters are taken from different values is described, and the sensitivity of the solution to the initial value is studied.The tail filter is used to optimize the decision function through the gradient descent method, filtering the observation error, and obtaining the initial field which is closer to the true value.

Keywords: shadowing filter，weather forecast

1. Introduction

With the continuous development and perfection of the system model, numerical weather forecasting is increasingly attributed to the initial value problem. A clear analysis of the solution state of the model is helpful to understand the close relationship between the initial value and predictability and to highlight the importance of proper processing of the initial observation. Through the discussion of the predictability problem, the differences between the results obtained before and after filtering are compared, which can provide example support for the effective filtering error of the trailing filter and is helpful for the further popularization and application of the trailing filter in data optimization. At the same time, the filtered observation value can be regarded as a true value in a certain sense, and the accurate prediction results can be obtained by the initial observation within a certain error range. for example, the lower bound of the maximum predictable time instead of the maximum predictable time can be finally obtained, thus improving the prediction efficiency.

The numerical weather forecast uses a computer to calculate the development of the atmosphere in the next few days based on the weather observation data provided. Since the time integration of an atmospheric model belongs to the initial value problem, the ability to make accurate forecasts requires not only that the computer operating model can realistically represent the atmosphere, but also that the initial conditions of the atmosphere are considered accurate. With the continuous development and improvement of the model, the accuracy requirements for the initial conditions are also increasing day by day. physicist bjerknes once attributed the weather forecast to the initial value problem. Lorenz pointed out: after a possible transition period, stable systems with constant or periodic orbits will have trajectories starting from different points very close to each other, that is to say, they are completely predictable. As a kind of unstable dynamical system, even when the model is perfect and the initial conditions are almost completely correct, for example, only the rounding error of the initial values is considered, the two solutions obtained by the integration of the same model are very different in a few weeks, which shows that their predictability is limited. The research on predictability is concerned with the uncertainty of numerical weather forecast, which is mainly divided into problems related to initial error and problems related to model error. the former is the main research object. this project is also based on the assumption of perfect model. Furthermore, mumu and others divided the predictability problem into three sub - problems: the lower bound of the maximum predictable time, the upper bound of the maximum prediction error, and the maximum allowable initial error and model error. The first two sub-problems consider the maximum prediction time that can be reached by a given prediction error and the maximum prediction error that can be reached by a given prediction time after the actual value is unknown but the error range between the given observation value and the actual value is taken as the initial field for numerical prediction. Since the true value is only included in the observation error range and the specific value is not known, the lower bound of the maximum predictable time can only be obtained by taking the minimum within the observation error range or the upper bound of the maximum predictive error can be obtained within the observation error range. As a method of filtering errors, Thomas stem ler and Kevin Judd [2] reveal that shadowing filter is processing the original observation data to improve the approximation degree between the original observation data and the true value, and finally achieve the effectiveness of improving the prediction effect。

2. Shadowing filters by gradient descent of indeterminism

Although the shadowing filter can be implemented in more general imperfect model situations, for the purposes of this discussion we will consider the perfect model scenariowith isotropic Gaussian noise. Consider a discrete time dynamical system on R^{d} with a dynamics given by the map $y_{t+1} = g(y_t)$. Assume $s_t = y_t + \xi$ is our observation of y_t, where ξ are independent Gaussian random variates with an isotropic variance σ^2. Also assume to have a model f of the system that is identical to g and that f is differentiable. The task of a shadowing filter is to find a sequence of states $X = (x_1, x_2 \cdots x_n)$ from a given sequence of observations $S = (s_1, s_2 \cdots s_n)$. X should be a trajectory of the model f and should shadow S. For X to be a trajectory requires $x_{i+1} = f(x_i)$, $i = 1, \cdots, n-1$. The trajectory will shadow S if the distances $\|s_i - x_i\|$ $i = 1, \cdots, n$ are not large relative to σ.

One method of implementing a shadowing filter is gradient descent of indeterminism (GDI). For any sequence of states X define the indeterminism:

Define for any sequence of X its indeterminism:

$$I(X) = \frac{1}{n-1}\sum_{i=1}^{n-1}\left\|x_{i+1} - f(x_i)\right\|^2 \quad (1)$$

.

The sequence of states X can be considered as a point in R^{nd}, and I(X) is a scalar function on this a (n×d)-dimensional space. Clearly,X will be a trajectory of the model if and only if I(X) = 0. The indeterminism should be interpreted as a measure of how far a sequence of states X is from being a trajectory. The squared norm $\|\cdot\|^2$ can be replaced by any appropriate metric. Using a squared norm does not imply any assumption about a Gaussian distribution mismatches or otherwise, see elsewhere [1] for further discussion of this point. For the observations S the indeterminism I(S) is almost surely non-zero. The gradient descent method takes the observations S as a starting sequence of states X, then follows the steepest descent of the gradient of I(X) down to a minimum where I(X) = 0. This is equivalent to solving the differential equations on R^{nd}: $\frac{dX}{d\tau} = -\nabla I(X(\tau))$, $X(0) = S$ .The GDI shadowing filter is the result of taking the limiting sequence X(τ) as $\tau \to \infty$.

A more practical method is to solve the differential equation by a Euler iteration until suitable convergence is achieved, which provides an iterative GDI shadowing filter. Let $X_0 = s$ and $X_m = (x_{1,m} \cdots x_{n,m})$, where

$$x_{i,m+1} = x_{i,m} - \frac{2\Delta}{n-1} \times \begin{cases} -A(x_{i,m})(x_{i+1,m} - f(x_{i,m})), i = 1 \\ x_{i,m} - f(x_{i-1,m}) - A(x_{i,m})(x_{i+1,m} - f(x_{i,m})), 1 < i < n \\ x_{i,m} - f(x_{i-1,m}), i = n \end{cases} \quad (2)$$

Here $A(x)$ denotes the adjoint of f (transpose of the Jacobian matrix) evaluated at x, and Δ is an arbitrary step size. Later on we outline a method to find suitable values for the step size Δ, but typically the choice $2\Delta/(n-1) = 0.1$ will lead to a convergence of the iterative GDI shadowing filter. Details on the properties of such GDI shadowing filters are given elsewhere [5] Here we just want to mention that the GDI method always converges to a shadowing trajectory of the model and I(X) converges monotonically to zero. Furthermore, given a long observation sequence with sufficiently small bounded measurement noise of a hyperbolic system it can be shown that for perfect models the GDI shadowing filter converges to the true trajectory. In practice we will iterate eq. (2) until Xm has converged sufficiently. The remaining magnitude Im is one quantity that measures the quality of the estimated states. In addition we define below three other quantities that measure the quality: The magnitude of mismatch In,m, the root mean

$$I_{n,m} = \left\|x_{n,m} - f(x_{n-1,m})\right\| \quad (3)$$

$$E_m = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left\|x_{i,m} - y_i\right\|^2} \quad (4)$$