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STUDY OF CONVERGENCE OF LAGUERRE WAVELET BASED NUMERICAL METHOD FOR INITIAL AND BOUNDARY VALUE BRATU-TYPE PROBLEMS

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Abstract. Based on Laguerre wavelets, an efficient numerical method is proposed for the numerical solution of initial and boundary value Bratu-type problems arising in fuel ignition of the combustion theory and heat transfer. Convergence of the method for these kinds of problems is addressed in the form of theorems with proof. To illustrate the ability of the method is validated on test problems, numerical results are compared with exact and those from existing methods in the literature. The results demonstrate the accuracy and the efficiency of the Laguerre wavelet based numerical method.

Keywords: Laguerre wavelet method, Bratu's problem, Convergence method, Limit point.

1. Introduction

Wavelet theory is a relatively new and an emerging area in mathematical research. Wavelets permit the accurate representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithms. The main advantage of using orthogonal basis is that it reduces the problem into solving a system of algebraic equations. In recent years, the wavelets are dealing with dynamic system problems, especially in solving differential equations with two-point boundary value constraints have been discussed in many papers [13, 19]. By transforming differential equations into algebraic equations, the solution may be found by determining the corresponding coefficients that satisfy the algebraic equations. Some efforts have been made to solve Bratu's problem by using the wavelet collocation method [19].

Nonlinear phenomena are of fundamental importance in various fields of science and engineering. The nonlinear models of real-life problems are still difficult to solve either analytically or numerically. Now a day's our attention devoted to the search for better and more efficient solution methods for determining a solution. We mention that the spectral collocation method is very useful in providing highly accurate solutions to nonlinear differential equations [11]. Here, we intend to extend the application of Laguerre wavelet method to solve nonlinear initial value problems and boundary value problems of Bratu type. To the best of our knowledge, there are no results on Laguerre wavelet approximation for Bratu type equations arising in mathematical physics. This partially motivated our interest in such method. The aim of this paper is to study convergence of Laguerre wavelet method for boundary and initial value Bratu-type problems [1, 18, 19].

It is well known that Bratu's boundary value problem in one-dimensional planar coordinates is of the form

$$y'' + \beta e^{y} = 0, \quad 0 < x < 1 \tag{1.1}$$

with boundary conditions $y(0) = y(1) = 0$. for $\beta > 0$ is constant, the exact solution of (1.1) is given by [5],

$$y(x) = -2ln\left[\frac{cosh\left(\frac{\theta(x-\frac{1}{2})}{2}\right)}{cosh\left(\frac{\theta}{2}\right)}\right] \tag{1.2}$$

where θ satisfies,

$$\theta = \sqrt{2\beta}sinh\left(\frac{\theta}{2}\right) \tag{1.3}$$

The problem has zero, one and two solutions when $\beta > \beta_c, \beta = \beta_c, and\ \beta < \beta_c$, respectively, where the critical value β_c satisfies the equation,

$$1 = \frac{1}{4}\sqrt{2\beta_c}cosh\left(\frac{\theta}{4}\right).$$

The critical value β_c is given by $\beta_c = 3.513830719$ [3, 7, 8].

In addition, an initial value problem of Bratu's type,

$$y'' + \beta e^y = 0, \quad 0 < x < 1 \tag{1.4}$$

with initial conditions $y(0) = y'(0) = 0$ will be investigated.

Applications of the Bratu type equations are employed in the fuel ignition model of the thermal combustion theory, the model of thermal reaction process, the Chandrasekhar model of the expansion of the universe, chemical reaction theory, radioactive heat transfer and nanotechnology [6]. A substantial amount of research work has been directed for the study of the Bratu problem [3, 6, 10, 17]. Several numerical techniques, such as the finite difference method, finite element approximation, weighted residual method, and the shooting method, have been implemented independently to handle the Bratu model. In addition, Boyd [6] employed Chebyshev polynomial expansions and the Gegenbauer as base functions. Syam and Hamdan [15] presented the Laplace Adomain decomposition method (LADM) for solving Bratu's problem.

In this paper, Laguerre wavelet based numerical method is presented for the approximate solution of Bratu's problem. The method is based on expanding the solution by Laguerre wavelets with unknown coefficients. The properties of Laguerre wavelets together with the collocation method are utilized to evaluate the unknown coefficients and then an approximate solution to eq. (1.1) is identified.

The organization of the rest of the paper is as follows. In section 2, properties of Laguerre wavelet are described. In section 3, formulation of the method based on Laguerre wavelet is defined for initial and boundary value Bratu-type problems. Analysis of the Laguerre wavelet method for Bratu-type problems is presented in section 4 in the form of theorems with proof. Numerical results are reported in section 5, and finally conclusions are drawn in section 6.

2. Properties of Laguerre wavelet

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter ***a*** and the translation parameter ***b*** varies continuously, we have the following family of continuous wavelets:

$$\psi_{a,b}(x) = |a|^{\frac{-1}{2}}\, \psi\left(\frac{x-b}{a}\right), a, b \in R, a \neq 0.$$

If we restrict the parameters ***a*** and ***b*** to discrete values as

$$a = a_0^{-k}, b = nb_0 a_0^{-k}, a_0 > 1, b_0 > 0,$$

we have the following family of discrete wavelets :

$$\psi_{k,n}(x) = |a_0|^{\frac{1}{2}}\, \psi\left(a_0^k x - nb_0\right).$$

Where $\psi_{k,n}$ form a wavelet basis for $L^2(R)$. In particular, when $a_0 = 2$ and $b_0 = 1$, then $\psi_{k,n}(x)$ forms an orthonormal basis.

The Laguerre wavelets $\psi_{k,n}(x) = \psi(k, n, m, x)$ involve four arguments *n =1, 2, 3,...,* 2^{k-1}, *k* is assumed any positive integer, *m* is the degree of the Laguerre polynomials and it is the Normalized time. They are defined on the interval [0, 1) as

$$\psi_{k,n}(x) = \begin{cases} 2^{\frac{k}{2}}\bar{L}_m(2^k x - 2n + 1), & \frac{n-1}{2^{k-1}} \le x < \frac{n}{2^{k-1}} \\ 0 & otherwise \end{cases} \tag{2.1}$$

$$\text{where } \bar{L}_m(x) = \frac{L_m}{m!} \tag{2.2}$$

m = 0, 1, 2... M-1. In eq. (1.2) the coefficients are used for orthonormality. Here $L_m(x)$ are the Laguerre polynomials of degree m with respect to the weight function W(x) =1 on the interval $[0,\infty)$ and satisfy the following recursive formula $L_0(x) = 1$, $L_1(x) = 1 - x$,

$$L_{m+2}(x) = \frac{(2m + 3 - x)L_{m+1}(x) - (m + 1)L_m(x)}{m + 2}, \qquad m = 0,1,2 \ldots$$

A function $y(x)$ defined over [0, 1) can be expanded as a laguerre wavelet series as follows: