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The primal-dual simplex algorithm base on the most obtuse angle principle

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Abstract We present a relaxation algorithm for solving linear programming (LP) problems under the framework of the primal-dual simplex algorithm. Each iteration, based on a heuristic representation of the optimal basis (the principle of the most obtuse Angle), the algorithm constructs and solves a sub-problem, whose objective function is the same as the original problem, but only contains partial constraints.The primal-dual simplex algorithm is used to solve the sub-problem. If the sub-problem has an optimal solution or the sub-problem has no feasible solution, the constraint is added according to the principle of the obtuse Angle and then the final solution of the old sub-problem is taken as the starting point. Our preliminary numerical experiments show that the proposed algorithm can effectively reduce the number of iterations compared with the traditional two-stage simplex algorithm. Iterations for sub-problems take up a significant proportion, which greatly reduces the CPU time required for each iteration. The new algorithm has potential advantages in solving large-scale problems. It's a very promising new algorithm.

Keywords: linear programming, primal-dual simplex algorithm, the most obtuse Angle principle, sub-problem

1. Introduction

Linear programming is an early and far-reaching branch of operational research. After more than 70 years of development, linear programming has been widely used in national economy, science and technology, management and engineering. It has produced enormous economic and social benefits.

In 1947, G.B.Dantzig[1,2]proposed the simplex algorithm for solving LP problems for the first time, which marked the establishment of this discipline. In 1979, Khachian[3], a young Soviet mathematician, proposed the first polynomial time complexity algorithm -- ellipsoid algorithm. It is proved that polynomial time algorithm exists in LP problem, but the actual test shows that its computational effect is far worse than simplex algorithm. Until 1984, Karmarkar[4] proposed another algorithm with polynomial time complexity for solving LP problems, Karmarkar inner point algorithm. Later experiments show that this algorithm not only has lower order polynomial time complexity than ellipsoid algorithm but also has very encouraging performance.

In 1990, professor Pan P.Q proposed the concept of principal element notation[5], and gave a heuristic representation of an optimal basis (the principle of the most obtuse Angle) ,which is used to further improve the computational efficiency of simplex algorithm.

In 1993, Konstantinos Paparrizos expounded a new simplex algorithm for solving LP problems, the external point simplex algorithm[7~9]. In these papers, a hybrid primal-dual simplex algorithm is formed by combining the exterior point simplex algorithm and the interior point algorithm.

In this paper, we will apply these good results to reduce the number of iterations and further improve the efficiency of simplex algorithm. First, we use the most obtuse angle principle to select the partial constraints in the LP problem to form the sub-problem. Then, we use the primal-dual algorithm to solve the problem. If the sub-problem has an optimal solution or no feasible solution, the constraint condition is added according to the obtuse Angle principle to update the sub-problem and then the final solution of the old sub-problem is taken as the starting point.

2. Construction of sub-problem

For LP problem, the fewer the decision variables, the fewer the constraint conditions, the easier the LP problem is to be solved.Therefore, this paper uses the most obtuse angle principle to form sub-problem to reduce the scale of LP problems[5,6].

For LP problems where the constraint condition is inequality:

$$\min z = c^T x \qquad s.t.\begin{cases} Ax \geq b \\ x \geq 0 \end{cases} \tag{2.1}$$

where $A \in R^{k\times n}$ with $k < n$ and $b \in R^k$, $c, x \in R^n$ 。It is assumed that the cost vector c , the right-hand side b , and A 's columns and rows are all nonzero. In addition, we stress that no assumption is made on the rank of A , except $1 \leq rank(A) \leq k$.

Partition matrix A by row: $A = \left(u_1^T, \cdots, u_k^T\right)^T$.

According to the most obtuse Angle principle , the active constraint is some constraints which form obtuse Angle with the objective function.Therefore,the primary dimension of LP problem is calculated by:

$$\alpha_i = u_i^T c (i = 1, \cdots, k) \tag{2.2}$$

$$\text{Set } J = \{i \mid \alpha_i \geq 0, i = 1, \cdots, n\}; , \quad |J| = m \tag{2.3}$$

Set $J_s = \{1, \cdots, k\} \setminus J$, then, only the constraints in J are retained and the remaining constraints are omitted to obtain a small scale sub-problem.

$$\min z = c^T x \qquad s.t.\begin{cases} A_J x \geq b_J \\ x \geq 0 \end{cases} \tag{2.4}$$

where $A_J \in R^{m\times n}$ and $b_J \in R^m$.

Since the simplex method of LP problem must be solved under the condition of equality, the relaxation variable is added in the constraint condition of LP problem (2.4), and the inequality is changed into the equation.

$$\min z = c^T x \qquad s.t.\begin{cases} A_J x - x_s = b_J \\ x, x_s \geq 0 \end{cases} \tag{2.5}$$

where x_s is the relaxation variable.

In this process, due to the introduction of the most obtuse Angle algorithm, the constraint of the sub-problem is reduced (compared with the original LP problem), which reduces the scale of the problem.

3. Primal-dual simplex method

When the LP sub-problem (2.5) is neither primal feasible nor dually feasible, the sub-problem is usually standardized and the initial basis is constructed by adding artificial variables. Then, the two-stage method is adopted to solve the problem. However, this method leads to the expansion of LP problem (2.5).In this paper, the primal-dual simplex method is used to solve the sub-problem[10~14].

Let us develop a tableau version of the LP problem first. Assume the presence of a canonical matrix, which might as well be denoted again by $\begin{pmatrix} B & N & b \end{pmatrix}$, with the associated sets J_B and J_N known, as partitioned:

$$\begin{pmatrix} A & b \\ c^T & 0 \end{pmatrix} \rightarrow \begin{pmatrix} B & N & b \\ c_B^T & c_N^T & 0 \end{pmatrix} \rightarrow \begin{pmatrix} I & B^{-1}N & B^{-1}b \\ 0 & \bar{z}_N^T & z \end{pmatrix} \tag{3.1}$$