ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 13, No. 3, 2018, *pp*.195-200

Primal-dual Simplex Algorithm for Linear Programming

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(Received March 03 2018, accepted August 19 2018)

Abstract. When the linear programming (LP) problem satisfies neither the primal feasibility nor the dual feasibility, the primal-dual simplex method can be used to solve the problem in order to avoid the defect of the increase of decision variables caused by the addition of artificial variables in the two-stage algorithm. In this paper, we use PPSM (primitive - primitive simplex method) and DDSM(dual - dual simplex method) to solve these problems. Compared with the simplex method of two-stages, a better result is obtained.

Keywords: Primal feasibility; Dual feasibility; PPSM; DDSM.

1. Introduction

Linear programming (LP) problem is an important branch of operational research. It studies Mathematical theories and methods of extremum problems under linear objective function and constraint [1].LP problem has been widely used in military operations, economic analysis, and operation management and engineering technology. Reasonable utilization of limited human, material and financial resources will provide a scientific basis for decision makers to make optimal decisions， which assists and guides people to conduct scientific management and planning.

In 1947, American mathematician G.B.Dantzig proposed the general mathematical model and classical algorithm of LP problem, simplex method [2, 3], which laid a foundation of the subject. The algorithm points out that the optimal solution of LP problem must be reached at a vertex (basic feasible solution) of the feasible domain if it exists, since the feasible domain of LP problem is convex set. Two methods can be used to construct the initial basic feasible solution, which are two-stage method and big M method [1]. That is, the initial base matrix can be constructed by adding artificial variables. But, they lead to the increase of decision variables of LP problem, the increase of scale and the increase of calculation quantity.

In 1972, V.Klee and G.J.Minty pointed out that simplex algorithm was not an algorithm of polynomial time in computational complexity. They illustrated by an example that simplex algorithm was exponential time in the worst case. In 1979, Khachian proposed ellipsoid algorithm [4], which proved that there is indeed polynomial time algorithm in LP problem, but the actual effect is not satisfactory. In 1984, Karmarkar proposed a new polynomial time algorithm, Karmarka interior point algorithm [5], which not only has lower order polynomial time complexity than ellipsoid algorithm, but also has encouraging practical performance. Subsequently, the internal point method is hot, and a number of good algorithms are produced. Many scholars believe that solving large-scale problems is better than simplex algorithm. However, it cannot be used to solve the integer LP problem because it cannot be started hot, and the interior point algorithm cannot shake the dominance of simplex algorithm in practice.

In 1993, Konstantinos Paparrizos expounded a new simplex algorithm for solving linear programming problems, the external point simplex algorithm [6-8]. Literature [8] describes the modified form of the external point simplex algorithm, and combines the outer point simplex algorithm with the interior point algorithm to form a hybrid primal-dual simplex algorithm. The algorithm forms two paths: one converges from the inside or the boundary of the feasible region to the optimal solution of the original problem, and the other from the outside of the feasible region to the optimal solution of the original problem. The algorithm extends the original feasible solution to any solution of the original problem.

In 1998, PAN P.Q extends the concept of base, puts forward a new kind of simplex algorithm, a basis-deficiency-allowing variation of the simplex method [9]. It does not require that the basis be square matrix in the iterative process, as long as the definition of generalized basis can be satisfied as the basis in the iteration. Of course, the selection rules of the incoming, outgoing base variables and the construction methods of the initial basic feasible solutions have been changed. In 2003, Pan proposed the two-stage simplex algorithm for the deficient basis [10, 11], which marked the improvement of the deficient basis theory. That is, there is a perfect description from the formation of the initial feasible basis, the selection of the incoming, outgoing basis, the optimal termination criteria and so on.

In this paper, for a LP problem which is neither primal feasible nor dual feasible, if it can construct a basic feasible solution of LP problem or the basic feasible solution of dual problem, the primal simplex method or dual simplex method can be used to solve LP problem.

2. The Generalization of Basis

We consider the following LP problem [12-15]:

$$\min z = c^T x$$
$$s.t.\begin{cases} Ax = b \\ x \geq 0 \end{cases} \tag{1}$$

where $A \in R^{m\times n}$ with $m < n$ and $b \in R^m$, $c, x \in R^n$. It is assumed that the cost vector c, the right-hand side b, and A's columns and rows are all nonzero. In addition, we stress that no assumption is made on the rank of A, except $1 \leq rank(A) \leq m$.

Let B be the initial basis matrix, $B \in R^{m\times m}$. Let N is a non-basis matrix, containing the remaining columns. Define the ordered basic and nonbasic (index) sets respectively by

$$J_B = \{j_1, \cdots, j_m\}, J_N = \{k_1, \cdots, k_n\} \tag{2}$$

where $j_i, i = 1, \cdots, m$, is the index of the i^{th} column of B, and $k_j, j = 1, \cdots, n$, is the index of the j^{th} column of N. The subscript of a basic index j_i is called row index, and that of a nonbasic index k_j column index.

Let us develop a tableau version of the LP problem first. Assume the presence of a canonical matrix, which might as well be denoted again by $\begin{pmatrix} B & N & b \end{pmatrix}$, with the associated sets J_B and J_N known, as partitioned:

$$\begin{pmatrix} A & b \\ c^T & 0 \end{pmatrix} \to \begin{pmatrix} B & N & b \\ c_B^T & c_N^T & 0 \end{pmatrix} \to \begin{pmatrix} I & B^{-1}N & B^{-1}b \\ 0 & \bar{z}_N^T & z \end{pmatrix} \tag{3}$$

where I is a unit matrix, z is the objective value. From (3), a solution can be determined immediately:

$$\begin{cases} x_B = B^{-1}b \\ x_N = 0 \end{cases} \tag{4}$$

Assume that the current canonical tableau, say (3).

If $\bar{b} \geq 0$, the basic solution is the basic feasible solution. If, in addition, it holds that $\bar{z}_N^T \geq 0$, the dual feasibility is already obtained, and hence all is done. If not, the LP problem can be solved by the primal simplex method.

When $\bar{b}$ is not all ≥ 0, LP problem does not satisfy the primal feasibility. If the test number of LP problem $\bar{z}_N^T \geq 0$, dual simplex method can be used to solve it. If not, the LP problem does not satisfy the dual feasibility. That is to say neither the primal simplex method nor the dual simplex method can deal with the problem.

In general, two-stage method is used to solve these problems, but it complicates the mathematical model and increases the computer time. To solve LP problem, the following two methods can be adopted [13-20].

Suppose $\bar{a}_j$ is the j^{th} column of $(I_B, B^{-1}N)$, $\bar{a}_{ij}$ represents the elements of i^{th} row and j^{th} column of the coefficient matrix.

Case 1: If the primal feasibility of LP problem can be satisfied, the solution of LP problem can be realized.

In this case, the row index p may be determined such that:

$$p = \arg\min\{\bar{b}_i \mid i = 1, \cdots, m\} \tag{5}$$

So, if $\bar{b}_p < 0$, the basic variable x_{j_p} leave the basis.

We select a basic column to enter the basis, as below:

$$q = \arg\min\{a_{pk_i} \mid i = 1, \cdots n\} \tag{6}$$