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Hopf bifurcation analysis in a predator-prey model with square root response function with two time delays

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Abstract. In this paper, we investigate the local stability and Hopf bifurcation analysis in a predator-prey model with square root response function and two time delays. By choosing the two delays as the bifurcation parameter and by analyzing the corresponding characteristic equations, the conditions for the stability and existence of Hopf bifurcation for the system are obtained. Finally, the corresponding numerical simulations are carried out to support the theoretical analysis.

Keywords: Hopf bifurcation, predator-prey model, square root response function, time delay

1. Introduction

The interaction between the predator and the prey is the fundamental structure in population dynamics. Predator-prey models have been widely researched in both ecology and mathematical ecology[1-7]. There has been a great and continuing interest in predator–prey models with time delay, functional response, etc. The effect of these factors has explained more richer and complicated dynamics.

Time delay is a natural phenomenon which exists universally and unavoidably in nature. In biological systems, such as the gestation process of species, the migration process of species, the digestion and transformation process of species after capture of prey and its own mature time, etc, they all belong to time delay phenomena[8-12]. In [8], Song et al. presented a predator–prey system with stage structure and time delay for the prey. In [10], Peng and Zhang considered a delayed predator-prey system with Holling type III functional response incorporating a prey refuge and selective harvesting. In [12], Hao et al. studied a diffusive single species model with stage structure and strong Allee effect subject to homogeneous Neumann boundary condition. From the discussions of above references, we found that time delay destroyed the stability of the system at the equilibrium point and caused the periodic fluctuations of the species, which can lead to various forms of bifurcation behavior or chaotic movement in the related systems. Therefore, in order to maintain the ecological balance, it is meaningful to fully study the factor of time delay.

In the predator-prey model, one of the important factors affecting population dynamics is functional response, which reflects the predatory ability of predators. In [13], Salmanet al. researched a discrete predator-prey system with square root functional response, they derived the flip and Niemark-Sacker bifurcations. In [14], Braza analyzed a predator-prey model with square root functional responses in which a modified Lotka–Volterra interaction term is used as the functional response of the predator to the prey. Based on the above considerations, in this paper, we shall investigate the dynamic analysis in a predator-prey model with two time delays and square root functional response.

The remainder of this paper is organized as follows. In Section 2, we present a general description for the predator-prey model. In Section 3, the local stability and the existence of Hopf bifurcation at the positive equilibrium are discussed. In Section 4, we perform some numerical simulations which are revealed to illustrate the validity of the theoretical results. Finally, making a brief conclusion in Section 5.

2. The delayed predator-prey model with square root response function

In [14], Braza proposed a predator–prey model with square root functional responses:

$$\dot{x}(t) = x(t) - x^2(t) - \sqrt{x(t)}y(t),$$
$$\dot{y}(t) = -sy(t) + c\sqrt{x(t)}y(t),$$

(1)

where $x(t)$ and $y(t)$ can be described as the population densities of prey and predator at time t, s denotes the death rate of the predator, and c is the biomass conversion or consumption rate. He studied the dynamics of the square root system and compared with the dynamics of predator–prey systems that used a typical Lotka–Volterra interaction term.

However, the factor about time delay often appeared in actual situation. In this paper, motivated by Braza [14]and Zhu et al. [15], we introduce two time delays and square root functional responses into the following predator-prey model:

$$\begin{aligned}\dot{x}(t) &= x(t) - x(t)x(t-\tau_1) - \sqrt{x(t)}y(t),\\ \dot{y}(t) &= -sy(t) + c\sqrt{x(t-\tau_2)}y(t),\end{aligned} \tag{2}$$

where the parameters $x(t), y(t), c, s$ are defined in system (1). τ_1 is the time delay due to the gestation of prey, and τ_2 is the feedback delay,that is to say, the predator takes the time to convert the food into its growth.

3. Local stability and Hopf bifurcation analysis

In this section, we shall discuss the stability of system (2) at the positive equilibrium and the existence of Hopf bifurcation by analyzing the corresponding linearized system.

It is obvious that system (2) has a unique positive equilibrium $E(x^*, y^*)$, where

$$x^* = \frac{s^2}{c^2},\ y^* = \frac{sc^2 - s^3}{c^3},$$

if the following condition (H1) $c > s$ satisfies.

Let $\bar{x}(t) = x(t) - x^*, \bar{y}(t) = y(t) - y^*$ and represent $\bar{x}(t), \bar{y}(t)$ by $x(t), y(t)$, respectively. Using Taylor expansion to expand the system (2) at the posotive equilibrium $E(x^*, y^*)$, then we can get the linearized system of system (2) as follows:

$$\dot{x}(t) = a_1x(t) + a_2x(t-\tau_1) + a_3y(t),$$
$$\dot{y}(t) = b_2x(t-\tau_2) + b_3y(t),$$

(3)

where

$$a_1 = 1 - x^* - \frac{y^*}{2\sqrt{x^*}},\ a_2 = -x^*,\ a_3 = -\sqrt{x^*},$$

$$b_2 = \frac{cy^*}{2\sqrt{x^*}},\ b_3 = -s + c\sqrt{x^*}.$$

The corresponding characteristic equation of system (3) is given by

$$\lambda^2 - (a_1 + b_3)\lambda + a_2(b_3 - \lambda)e^{-\lambda\tau_1} - a_3b_2e^{-\lambda\tau_2} + a_1b_3 = 0.$$

(4)

In order to investigate the distribution of roots of the transcendental equation (4), the result of Ruan and Wei [16] is introduced here.

Lemma 1 For the transcendental equation

$$\begin{aligned}p(\lambda, e^{-\lambda\tau_1}, \ldots, e^{-\lambda\tau_m}) = {}& \lambda^n + p_1^{(0)}\lambda^{n-1} + \cdots + p_{n-1}^{(0)}\lambda + p_n^{(0)}\\ &+ [p_1^{(1)}\lambda^{n-1} + \cdots + p_{n-1}^{(1)}\lambda + p_n^{(1)}]e^{-\lambda\tau_1} + \cdots\\ &+ [p_1^{(m)}\lambda^{n-1} + \cdots + p_{n-1}^{(m)}\lambda + p_n^{(m)}]e^{-\lambda\tau_m} = 0.\end{aligned}$$