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Confidence ellipsoids for the primary regression coefficients in m-equation seemingly unrelated regression models

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Abstract. For a m-equation seemingly unrelated regression(SUR) model, this paper derives two basic confidence ellipsoids(CEs) respectively based on the two-stage estimation and maximum likelihood estimation(MLE), and corrects the two CEs using the Bartlett correction method, resulting in four new CEs. In the meantime via using the partition matrix, we derive a new matrix-derivative-based formulation of Fisher's information matrix for calculating the maximum likelihood estimator of the m-equation SUR model. By Monte Carlo simulation, the coverage probabilities and average volumetric characteristics of CEs are compared under different sample values and different correlation coefficients. Moreover, it is proved that the CE based on the second bartlett correction method performs better even in the case of small samples. Finally, we apply these CEs to the actual data for analysis. The CEs of the SUR model with multiple equations are found to be more accurate than the case with only two equations.

Keywords: Seemingly unrelated regression, Confidence ellipsoids, Bartlett correction, Maximum likelihood estimation.

1. Introduction

The seemingly unrelated regressions (SUR) model, proposed by Arnold Zellner[1][2] in 1962, is a statistical model that is adopted broadly in various fields, such as economics, finance, medicine, biology and so on. In nature, the SUR model is a generalization of a linear regression model and consists of several regression equations with correlated error terms. The research on such models mainly uses the correlation of errors as additional information to improve the estimates efficiency of regression coefficients.

Zellner[2] used a generalized least squares approach to solve the regression coefficients of the SUR model. Moreover, a Bayesian estimation approach for this model was also introduced firstly by Zellner in[3]. Revankar[4][5] has proved properties of estimators via using residuals as estimates of the covariance matrix. V.K.Srivastava and A.K.Srivastava[6] have raised an improved estimation which is a convex combination of the ordinary-least-squares(OLS) estimator and a SUR estimator. In addition, V.Srivastava and D.Giles[7] have summarized kinds of estimation approaches and theories of SUR model at that time. Liu[8] has discussed two-stage estimators of the SUR model when the error terms defer to a general elliptical distribution. Zhao and Xu[9] have studied the high dimensional seemingly unrelated regression models recently.

In addition to OLS estimation and Bayesian estimation, scholars also studied another point estimation method - MLE approach. Phillips[10] has derived the exact distribution of the SUR estimators, which distribution is too complicated to use in practice. Thus, many scholars tried to use the approximate method to replace it. Park[11] has determined the suitable SUR-based MLE via using an iterative procedure. What's more, Fraser et al.[12] have analyzed the highly accurate likelihood method for the SUR model. However, this procedure is used only for scalar parameter and not for vector parameters.

Point estimate provides specific estimates of unknown parameters, which is easy to calculate and apply. However, its accuracy needs to be reflected by its distribution. In fact, interval estimation is the most intuitive way to measure the accuracy of a point estimate. In statistics, the confidence region is a multi-dimensional summary of confidence intervals. It is a set of points in n-dimensional space whose shape is usually an ellipsoid around a point. L. Le. Cam[13] has proposed the standard asymptotic confidence ellipsoids of Wald based on Hellinger distances. Lee. C. Adkins and R. Carter. Hill[14] have raised an improved confidence ellipsoid of the Stein-rule estimator for the linear regression model by bootstrap

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method. Laurent El Ghaoui and Giuseppe Calafiore[15] have given a method of computing confidence ellipsoids for uncertain linear equations with structure. Erik Weyer and M.C.Campi[16] have considered the finite sample properties of least-squares system identification and have derived non-asymptotic confidence ellipsoids for the OLS estimator. However, this paper considers the CEs of the SUR model.

In recent years, the research on two equations of SUR model has made great progress in parameter estimation and statistical inference. However, there are relatively few studies and applications of the SUR model of $m(m>2)$ equations, especially for the study of confidence region of the estimator. Kent R.Riggs et al.[17] have learned confidence ellipsoids for the primary regression coefficients in two seemingly unrelated regression models and have advanced two new confidence ellipsoids. This paper mainly studies the parameter estimation of multiple equations of SUR model and the confidence region of the estimator. Here we considered using two-stage estimation method to estimate the unknown parameters. Meanwhile, according to the MLE covariant parameter method of the SUR model derived from the two equations of Kent R.Riggs et al, we generalize it to m equations of the SUR model, and obtain the MLE estimated covariant by using Fisher's information matrix. By using these two kinds of estimators, Wald statistics are constructed to obtain the corresponding CEs, and then the accuracy of the CEs are modified according to two bartlett correction methods.

Using Monte Carlo simulation, we find that the cover probability of the second modified CE is close to the theoretical value and the accuracy of the second modified CE is similar to that of the other two corresponding ellipsoid in the small sample size. Specifically, the CEs from MLE perform better basically than those which CEs from two-stage estimator in small sample size. At the same time, as the sample size increases, the coverage probabilities and accuracy of CEs increase accordingly. Furthermore, as ρ increases, the coverage probabilities of the CEs increase while the average volumes decrease.

The context of this paper is as follows. In section 2 we give the expression of m-equation SUR model. Then we list some of the commonly used estimators for this model, such as two-stage estimator and MLE in section 3. In section 4 we derive six different CEs via using Wald statistic and Slutsky's Theorem. In section 5 we examine properties of these CEs by using Monte Carlo simulation. In section 6 we use CEs estimation approaches to model a real data set and compare the accuracy under different number of equations. Also, we sum up some simple conclusions in section 7. Finally, we show generalize a matrix-derivative-based formulation of Fisher's information matrix to m equations of the SUR model in the Appendix.

2. The m-equation SUR model

Consider a m-equation SUR model

$$Y_i = X_i\beta_i + \varepsilon_i, i = 1,2,\cdots,m, \tag{1}$$

where $Y_i \in R_{n\times 1}$, $X_i \in R_{n\times p_i}$ with $rank(X_i) = p_i$, $\beta_i \in R_{p_i\times 1}$, $\varepsilon_i \in R_{n\times 1}$ such that $E(\varepsilon_i) = 0$, $COV(\varepsilon_i, \varepsilon_j) = \sigma_{i,j} I_n$, $i, j = 1,2,\cdots,m$, where $n_i > p$ and I_n is the identity matrix with the order n. Let

$$Y \equiv \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix},\ X \equiv \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_m \end{bmatrix},\ \beta \equiv \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix},\ \varepsilon \equiv \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix}.$$

Then, the model (1) can be rewritten as

$$Y = X\beta + \varepsilon \tag{2}$$

Let

$$\Sigma \equiv \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,m} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\mathrm{m},1} & \sigma_{\mathrm{m},2} & \cdots & \sigma_{\mathrm{m},m} \end{pmatrix} = \begin{pmatrix} \sigma_{1,1} & \Sigma_{1,2} \\ \Sigma_{2,1} & \Sigma_{2,2} \end{pmatrix}, \tag{3}$$