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THE GROWTH OF GENERALIZED ITERATED ENTIRE FUNCTIONS –I

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Abstract. In this paper, we study the generalized iteration of entire functions and investigate the growth of iterated entire functions of finite iterated order. Here we prove some results on the growth of iterated entire functions of finite iterated order. The results improve and generalize some earlier results.

Keywords: entire functions, growth, iteration.

1. Introduction, Definitions and Notation

In order to study the growth properties of generalized iterated entire functions, it is very much necessary to mention some relevant notations and definitions. For standard notations and definitions we refer to [4].

Notation 1.1. [10] Let and for positive integer *m*, $\log^{[0]} x = x$, $\exp^{[0]} = x$ and

$$\log^{[m]} x = \log(\log^{[m-1]} x), \quad \exp^{[m]} x = \exp(\exp^{[m-1]} x).$$

Definition 1.2. The order ρ_f and lower order λ_f of a meromorphic function *f* is defined as

$$\rho_f = \limsup_{r\to\infty} \frac{\log T(r,f)}{\log r}$$

and

$$\lambda_f = \liminf_{r\to\infty} \frac{\log T(r,f)}{\log r}.$$

If *f(z)* is entire then

$$\rho_f = \limsup_{r\to\infty} \frac{\log\log M(r,f)}{\log r}$$

and

$$\lambda_f = \liminf_{r\to\infty} \frac{\log\log M(r,f)}{\log r}.$$

Definition 1.3. The hyper order ρ_f and hyper lower order λ_f of a meromorphic function *f* is defined as

$$\bar{\rho}_f = \limsup_{r\to\infty} \frac{\log\log T(r,f)}{\log r}$$

and

$$\bar{\lambda}_f = \liminf_{r\to\infty} \frac{\log\log T(r,f)}{\log r}.$$

If *f(z)* is entire then

$$\bar{\rho}_f = \limsup_{r\to\infty} \frac{\log^{[3]} M(r,f)}{\log r}$$

and

$$\bar{\lambda}_f = \liminf_{r\to\infty} \frac{\log^{[3]} M(r,f)}{\log r}.$$

Definition 1.4. [6] A function A function $\lambda_f(r)$ is called a lower proximate order of a meromorphic function *f* if

(i) $\lambda_f(r)$ is nonnegative and continuous for $r \ge r_0$, say;

(ii) $\lambda_f(r)$ is differentiable for $r \ge r_0$ except possibly at isolated points at which $\lambda'_f(r-0)$ and $\lambda'_f(r+0)$ exist;

(iii) $\lim_{r\to\infty} \lambda_f(r) = \lambda_f < \infty$;

(iv) $\lim_{r\to\infty} r\lambda'_f(r)\log r = 0$; and

(v) $\liminf_{r\to\infty} \frac{T(r,f)}{r^{\lambda_f(r)}} = 1$.

Definition 1.5. [6] Let $f(z)$ *and* $g(z)$ are two entire functions defined in the open complex plane and $\alpha \in (0,1]$. Then the generalized iterations of *f* with respect to *g* is defined as follows:

$$f_{1,g}(z) = (1-\alpha)z + \alpha f(z)$$

$$f_{2,g}(z) = (1-\alpha)g_{1,f}(z) + \alpha f\left(g_{1,f}(z)\right)$$

$$f_{3,g}(z) = (1-\alpha)g_{2,f}(z) + \alpha f\left(g_{2,f}(z)\right)$$

$$\cdots\cdots\quad \cdots\cdots\quad \cdots\cdots\quad \cdots\cdots$$

$$\cdots\cdots\quad \cdots\cdots\quad \cdots\cdots\quad \cdots\cdots$$

$$f_{n,g}(z) = (1-\alpha)g_{n-1,f}(z) + \alpha f\left(g_{n-1,f}(z)\right)$$

and so