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On solving fuzzy matrix games via linear programming approach

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Abstract In this paper, a two person zero- sum matrix game with $L-R$ fuzzy numbers payoff is introduced. Using the fuzzy number comparison introduced by Rouben's method, 1996, the fuzzy payoff is converted into the corresponding crisp payoff. Then, For each player, a linear programming problem is formulated. Also, a solution procedure for solving each problem is proposed. Finally, a numerical example is given for illustration.

Keywords: Matrix games; $L-R$ fuzzy numbers payoff; Optimal fuzzy strategy

1. Introduction

Game theory is concerned with decision making problem where two or more autonomous decision makers have conflicting interests. They are usually referred to as players who act strategically to find out a compromise solution (Kumar (2016)) . Zero- sum games refer to pure conflict. The payoff of one player is the negative of the payoff of the other player. Peski (2008) compared the structure information in zero- sum games.

As known, fuzzy set theory was introduced by Zadeh (1965) to deal with fuzziness. Up to now, fuzzy set theory has been applied to broad fields. Fuzzy set theory introduced by Zadeh (1965) make a model has to be set up using data which is approximately known. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers). For the fuzzy set theory development, we may referee to the papers of Kaufmann(1975), and Dubois and Prade(1980), they extended the use of algebraic operations of real numbers to fuzzy numbers by the use a fuzzifaction principle. Fuzzy linear constraints with fuzzy numbers were studied by Dubois and Prade (1980). In real- world problems, uncertainties may be estimated as intervals, Shaocheng 1994 studied two kinds of linear programming problems with fuzzy numbers called: interval numbers and fuzzy number linear programming, respectively. Tanaka et al. 1984 have formulated and proposed a method for solving fuzzy coefficients linear programming. Bellman and Zadeh (1970) introduced the concept of a maximizing decision making problem. Zhao et al. (1992) introduced the complete solution set for the fuzzy linear programming problems using linear and nonlinear membership functions.

Campos (1989) solved fuzzy matrix game using fuzzy linear programming. Cevikel and A hlatcioglu (2010) introduced new concept of solution for multi- objective two person zero- sum games. Xu (1998) discussed two- person zero- sum game with grey number payoff matrix. Dhingra et al. (1995) introduced a new optimization method to herein as cooperative fuzzy games and also solved the multiple objective optimization problems based on a proposed computational technique. An innovative fuzzy logic approach to analyze $n-$ person cooperative games is proposed by Espin et al.(2007). Xu and Yao (2010) studied rough payoff matrix games. Ein- Dor and Kantar (2001) and Takahashi (2008) discussed two- person zero- sum games with random payoffs. Applications of game theory may be found in economics, engineering, biology, and in many other fields. Three major classes of games are matrix games, continuous static games, and differential games. In continuous static games, the decision possibilities need not be discrete, and the decisions and costs are related in a continuous rather than a discrete manner. The game is static in the sense that no time history is involved in the relationship between costs and decisions. Elshafei (2007) introduced

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an interactive approach for solving Nash Cooperative Continuous Static Games, and also determined the stability set of the first kind corresponding to the obtained compromise solution. Khalifa and ZeinEldein (2015) introduced an interactive approach for solving cooperative continuous static games with fuzzy parameters in the objective function coefficients. Navidi et al. (2014) presented a new game theoretic- based approach for multi response optimization problem. Osman et al. (2015) introduced a new procedure for continuous time open loop stackelberg differential game. Roy et al. (2000) solved linear multiobjective programming based on cooperative game approach.

The remainder of the paper is as: In section 2., some preliminaries need in the paper are presented. In section 3, a two person zero- sum matrix game with fuzzy payoff is defined. In section 4, a solution procedure for solving the problems is introduced. A numerical example is given for illustration in section5. Finally some concluding remarks are reported in section 6.

2. Preliminaries

In order discuss our problem conveniently, we introduce fuzzy numbers and some of the results of applying fuzzy arithmetic on them and also comparison of fuzzy numbers by Roubens' method (Kauffmann and Gupta(1988); Fttemps and Roubens (1996)).

Definition1. A fuzzy number $\tilde{a}$ is a mapping defined as:

$\mu_{\tilde{a}} : R \to [0, 1]$, with the following:

(i) $\mu_{\tilde{a}}(x)$ is an upper semi- continuous membership function;

(ii) $\tilde{a}$ is a convex fuzzy set, i. e., $\mu_{\tilde{a}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)\}$, for all $x, y \in R, 0 \leq \lambda \leq 1$;

(iii) $\tilde{a}$ is normal, i. e., $\exists x_0 \in R$ for which $\mu_{\tilde{a}}(x_0) = 1$;

(iv) Supp $(\tilde{a}) = \{x \in R : \mu_{\tilde{a}}(x) > 0\}$ is the support of the $\tilde{a}$, and its closure cl (supp $(\tilde{a})$) is compact set.

Definition2. The $\alpha-$ level set of the fuzzy number $\tilde{a}$, , denoted by $(\tilde{a})_\alpha$ and is defined as the ordinary set:

$$(\tilde{a})_\alpha = \begin{cases} \{x \in R : \mu_{\tilde{a}}(x) \geq \alpha, \ 0 < \alpha \leq 1 \\ cl(\sup p(\tilde{a})), \qquad \alpha = 0 \end{cases}$$

A function, usually denoted by " L " or " R ", is a reference function of a fuzzy number if and only if

1. $L(x) = L(-x)$,
2. $L(0) = 1$,
3. L is nonincreasing on $[0, -\infty[$.

A convenient representation of fuzzy numbers in the $L-R$ flat fuzzy number which is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} L\big((A^- - x)\eta\big), & if\ x \leq A^-, \eta > 0, \\ R\big((x - A^+)\beta\big), & if\ x \geq A^+,\ \beta > 0, \\ 1, & eleswhere \end{cases}$$

where, $A^- < A^+, \left[A^-, A^+\right]$ is the core of $\tilde{A}$, $\mu_{\tilde{A}}(x) = 1; \forall x \in \left[A^-, A^+\right]$, A^-, A^+ are the lower and upper modal values of $\tilde{A}$, and $\eta > 0, \beta > 0$ are the left- hand and right- hand spreads (Roubens (1991)).

Remark1. A flat fuzzy number is denoted by $\tilde{A} = (A^-, A^+, \eta, \beta)_{LR}$