

Projective Synchronization of a Hyperchaotic Lorenz System

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Abstract: In this paper, the dynamical behaviors and projective synchronization of a five-dimensional hyperchaotic Lorenz system are investigated. First of all, a hyperchaotic system is constructed by introducing two state variables into the Lorenz chaotic system. Secondly, the dynamical behaviors of the proposed system, such as the dissipative property and equilibrium point, are discussed. Thirdly, based on the stability theory, the projective synchronization of the systems can be achieved. Finally, some numerical simulations are given to verify the projective synchronization scheme.

Keywords: Lorenz system, Hyperchaotic, Projective synchronization

1 Introduction

Chaos is a very interesting nonlinear phenomenon. In 1963, Lorenz discovered the famous Lorenz chaotic system [1]. After that, chaotic systems have been researched extensively, such as the Lü system [2-4], the Chen system [5] and the Rössler system [6]. Recently, much work has been done in constructing hyperchaotic models [7-9]. However, there is no universal method to get hyperchaotic systems. Compared with chaotic systems, hyperchaotic systems must have at least two positive Lyapunov exponents, and the dimension must be four or more [10]. Hyperchaotic systems can be obtained by adding one or more state variables to a three-dimensional chaotic system [11, 12]. Hyperchaotic systems have more abundant dynamical characteristics and complex behaviors than chaotic systems [13, 14]. So they are better suitable for some engineering applications, such as chemical reactions, electric circuits [15], cryptography [16, 17], fluid dynamics and secure communication [18-20].

Chaos synchronization is another fascinating concept. Pecora and Carroll proposed a drive-response chaotic synchronization scheme in 1990 [21], and realized the synchronization of two chaotic systems in the circuit, which promoted the theoretical study of chaotic synchronization and chaos control. Since then, many effective chaotic synchronization methods have emerged, such as complete synchronization [22, 23], generalized synchronization [24], phase synchronization [25], lag synchronization [26, 27], projective synchronization [28], anticipating synchronization [29] and exponential synchronization [30]. In recent years, the synchronization of chaotic fractional differential systems [31, 32] has attracted more and more attention because of its potential applications in secure communication and control processing [33-35].

The research of projection synchronization has received extensive attention from experts at home and abroad in recent years. Projection synchronization is that under certain conditions, the output of the coupled drive system and the response system state is not only phase locked, but the amplitude of each corresponding state also evolves according to a certain scale factor relationship. The method has been widely observed and discussed in coupled integer order chaotic systems.

The other parts of article is organized as follows. In section 2, a new five-dimensional hyperchaotic Lorenz system is constructed and the dynamical behaviors of the hyperchaotic system are discussed, such as attractor, dissipativity and equilibrium point. In section 3, the projective synchronization scheme of the hyperchaotic system is designed and some numerical simulations are completed. In section 4, some conclusions are given.

2 System description

2.1 A new hyperchaotic Lorenz system

The famous Lorenz chaotic system can be represented by the following autonomous differential equations

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = cx_1 - x_2 - x_1x_3, \\ \dot{x}_3 = x_1x_2 - bx_3 \end{cases} \quad (1)$$

where a, b and c are real constants. When the parameters are chosen as $a = 10, b = 8/3$ and $c = 28$, the system (1) is chaotic.

A new five-dimensional system is constructed by adding two variables into Lorenz chaotic system. In the first equation of the system (1), x_4 is introduced and the rate of change of x_4 is $\dot{x}_4 = -x_2x_3 + dx_4$. In the second equation of the system (1), another state x_5 is introduced and the rate of change of x_5 is $\dot{x}_5 = rx_1$. The new five-dimensional system can be described as

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_4 \\ \dot{x}_2 = cx_1 - x_2 - x_1x_3 + x_5 \\ \dot{x}_3 = x_1x_2 - bx_3 \\ \dot{x}_4 = -x_2x_3 + dx_4 \\ \dot{x}_5 = rx_1 \end{cases}, \quad (2)$$

where a, b, c, d and r are real constants. When the parameters are chosen as $a = 10, b = 8/3, c = 28, d = -6$ and $r = -5$, the system (2) is hyperchaotic. The chaotic attractors of the system (2) are plotted in Fig. 1 with the initial state $(x_1, x_2, x_3, x_4, x_5) = (1, 1, 1, 1, 1)$.

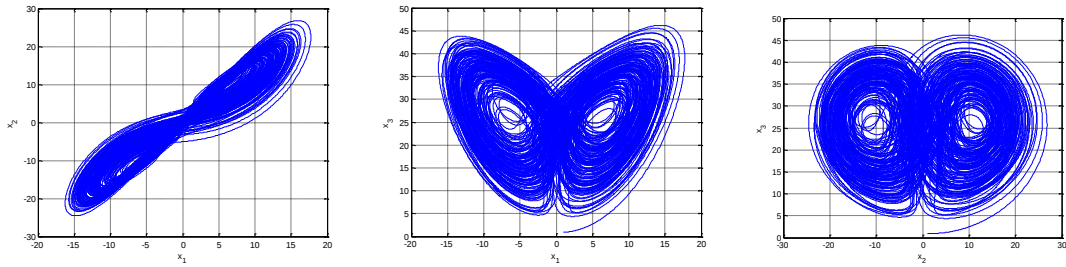


Fig. 1. Chaotic attractors of the system (2) in 2D spaces with $a = 10, b = 8/3, c = 28, d = -6$ and $r = -5$.

2.2 Dissipativity

The divergence of system (2) is calculated as

$$\nabla v = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} + \frac{\partial \dot{x}_4}{\partial x_4} + \frac{\partial \dot{x}_5}{\partial x_5} = -19.667. \quad (3)$$

When $\nabla v < 0$, the system (2) is a dissipative system and the exponential shrinkage is -19.667. That is, in the dynamical system (2), when $t \rightarrow +\infty$, each volume containing the dynamical system trajectory shrinks to zero at an exponential rate of -19.667. The orbit of the dynamical system is ultimately limited to a specific subset of zero volume, and the asymptotic motion is located on the attractors of the system (2).

2.3 Equilibrium point and stability

Let

$$\begin{cases} a(x_2 - x_1) + x_4 = 0 \\ cx_1 - x_2 - x_1x_3 + x_5 = 0 \\ x_1x_2 - bx_3 = 0 \\ -x_2x_3 + dx_4 = 0 \\ rx_1 = 0 \end{cases}. \quad (4)$$

The only equilibrium point $E_0(0, 0, 0, 0, 0)$ of system (2) is available. Then the Jacobian matrix of the system (2) at the equilibrium point E_0 is described as