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The perturbed compound Poisson-Geometric risk model with constant interest and a threshold dividend strategy

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Abstract: In this paper, the perturbed compound Poisson-Geometric risk model with constant interest and a threshold dividend strategy are considered. Firstly, the integro-differential equations with boundary conditions for the Gerber-Shiu function is discussed. Then the equation satisfying the ruin probability studied when the claim size is exponential function. Finally, Integro-differential equations with certain boundary for the moment-generation function of the present value of total dividends until ruin is derived.

Keywords: Constant interest; Threshold dividend strategy; Gerber-Shiu discounted penalty function; Integro-differential equation.

1. Introduction

Insurance company is a financial institution that operates risk business, whose operating condition is uncertain. For this reason, scholars have proposed some indicators to describe the operating conditions. The so-called bankruptcy of a company in the mathematical model refers to the probability of negative earnings in a certain period. If some factors affecting the surplus are taken into consideration, the insurance company may obtain a profit. The study of the insolvency theory can be traced back to the doctoral thesis published by Filip Lundbery in 1903[1], who proposed a class of random processes for the first time, that is, the Poisson process, and then Harald Cramer improved the results of Filip Lundberg and developed a strict stochastic process theory. Therefore, the improved model is called a CLASSICAL risk one. Thus, the surplus of an insurance company at time t can be given as

$$U(t) = u + ct - \sum_{t=1}^{N(t)} X_i \, , t \geq 0.$$

There are many conclusions in the classical model [2]. With the deep research of this model, many scholars have improved the classic risk model from different aspects. For example, the interference term or the random factor is added to the model [3, 4]. As we all know, the risk event is equivalent to the actual claim one in the classic risk model. The process which describes the number of claims is a homogeneous Poisson process. In fact, there is a deviation between the number of risk events and the actual claims. Therefore, a kind of composite Poisson-Geometric process is introduced, which is called PG Process [5-6]. The surplus of a Poisson-Geometric risk model at time t can be described as

$$U(t) = u + ct - \sum_{t=1}^{N(t)} X_i + \sigma W(t) \, , t \geq 0, \tag{1.1}$$

where $u \geq 0$ is the initial surplus; $c > 0$is the constant rate of premium; $\{X_i, i = 1,2, \cdots\}$ is a sequence of independent income size random variables with a common distribution function F (x) which satisfies F (0) = 0 and has density function f (x). $\{N(t) \geq 0, t \geq 0\}$is the Poisson-Geometric income-number process; $\{W(t), t \geq 0\}$ is a standard Brownian motion; $\sigma > 0$ is a constant representing diffusion volatility parameter.$\{X_i, i = 1,2, \cdots\}$, $\{N(t) \geq 0, t \geq 0\}$ and $\{W(t), t \geq 0\}$ are mutually independent.

Suppose that the insurer could receive interest form its surplus of (1.1) at a constant force of interest $r > 0$, then the surplus of the insurer at time t is

$$U(t) = ue^{rt} + c\int_0^t e^{r(t-s)} ds - \sum_{t=1}^{N(t)} e^{r(t-S_i)} X_i + \sigma \int_0^t e^{r(t-s)} \, dW(s), t \geq 0, \tag{1.2}$$

where S_i is the inter-time of the ith claim. Then (1.2) also can be rewritten as

$$U(t) = ue^{rt} + \frac{c}{r}(e^{rt} - 1) - \sum_{t=1}^{N(t)} e^{r(t-S_i)} X_i + \sigma \int_0^t e^{r(t-s)} \, dW(s), t \geq 0. \tag{1.3}$$

Gerber and Landry considered its expected discounted value of a penalty that is due at ruin [7]. With the development of the industry, the issue of dividend strategies has received remarkable attention since De Finetti first proposed the so-called barrier strategy to reflect the surplus cash flowed in an insurance portfolio [8]. Classical risk model with constant interest and a threshold dividend strategy were discussed [9, 10]. The expected discounted dividends before ruin under threshold-type dividend strategy was analyzed [11]. The constant barrier for the compound Poisson-Geometric risk model was studied in [12].

In this paper, we consider the modification of the surplus process by a threshold strategy with a threshold level $b(b > 0)$. when U (b) is below b, the surplus decreases at the original rate c_1; when U (b) is above b, the surplus decreases at a different rate $c_2(c_2 < c_1)$ and dividends are paid at rate $c_1 - c_2$. Incorporating the threshold strategy into (1.3) yields the surplus process $U_b(t)$, $t \geq 0$ which can be expressed by

$$dU_b(t) = \begin{cases} c_1\mathrm{dt} + \mathrm{rU}_b(t)\mathrm{dt} - \mathrm{dS}(t) + \sigma\mathrm{dW}(t) & \text{if } \mathrm{U}_b(t) < \mathrm{b} \\ c_2\mathrm{dt} + \mathrm{rU}_b(t)\mathrm{dt} - \mathrm{dS}(t) + \sigma\mathrm{dW}(t) & \text{if } \mathrm{U}_b(t) \geq b' \end{cases} \tag{1.4}$$

where $S(t) = \sum_{i=1}^{N(t)} X_i$, $U_b(0) = u$.

Let $T_b = \inf\{t \geq 0, U_b(t) \leq 0\}$ be the time of ruin, the Gerber-Shiu discounted penalty function is

$$\varphi(u,b) = E\left[e^{-\delta T_b} w(U_b(T-), |U_b(T)|) I(T_b < \infty) | U_b(t) = u\right],$$

where δ is a nonnegative parameter, $w(x,y)$is a nonnegative bounded measurable function of $(0,\infty) \times (0,\infty)$, and I(A) is an indicator function.

Let D(t) denote the cumulative amound of dividends paid out up to time t and β be the force of interest, then the present value of all dividends until T_b is

$$D_{u,b} = \int_0^{T_b(t)} e^{-\beta t} dD(t),$$

where $T_b(t) = \inf\{t \geq 0: U_b(t) \leq 0\}$ is the time of ruin. An alternative expression for $D_{u,b}$ is

$$D_{u,b} = (c_1 - c_2) \int_0^{T_b(t)} e^{-\beta t} I(U_b(t) > b)\, dt.$$

In the sequel, we are interested in the following moment generating function

$$M(u,y,b) = E[e^{yD_{u,b}}]$$

and the nth moment function

$$V_n(u,b) = E\left[D_{u,b}^n\right], for\ V_0(u,b) = 1.$$

2 Gerber-Shiu discounted penalty function

Firstly, the compound Poisson-Geometric process and its properties are introduced as Definition 2.1 and Definition 2.2.

Definition 2.1[5] If the probability generating function of random variable ζ is

$$G(t) = exp\frac{\lambda(t-1)}{1-\rho t},$$

then ζ has the compound Poisson-Geometric distribution as PG(λ, ρ).

Definition 2.2[5] N(t) $(t \geq 0)$ is called a compound Poisson-Geometric process with parameter $\lambda > 0$ and $0 \leq \rho < 1$, if the conditions

(1) N $(0) = 0$,

(2) N(t) $(t \geq 0)$ has has stationary and independent increments,

(3) For $t > 0$, N (t) has the PG(λ, ρ) distribution, and

$$EN(t) = \frac{t}{1-\rho}, VarN(t) = \frac{(1+\rho)}{(1-\rho)^2}$$

are satisfied.

Remark 2.1 In definition 2.2, ρ is defined as deviation parameter and describes the difference between risk event numbers and claim event numbers. When $\rho = 0$, the compound Poisson-Geometric process is the Poisson process. So the Poisson-Geometric process is a generalization of the Poisson process.

Lemma 2.1 N (t) is a process of a compound Poisson-Geometric process with parameter λ, ρ. Let $\alpha = \frac{\lambda(1-\rho)}{\rho}$, if $\rho = 0, \alpha = \lambda$, then we can have that when $t \to 0$,

$$\mathrm{P}(N(t) = 0) = e^{-\lambda t} = 1 - \lambda + o(t),$$

$$\mathrm{P}(N(t) = k) = \alpha\rho^k t + A_k(t)o(t), k = 1,2,...,$$