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Energy Dependence of Modified Hindmarsh-Rose Neuron under Periodic Disturbance

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Abstract: Due to the complex environment which neurons are located in, Hindmarsh-Rose neuron model is modified by introducing a new variable and linear coupling. Via numerical simulations, multiple modes of electric activities of the addressed neuron model can be observed by changing external forcing current. To explore the energy transition between different electric activity modes, Hamilton energy is calculated when the proposed neuron is disturbed by periodic signal. It is found that the energy is closely dependent on the electric activity mode caused by external forcing current. An interesting phenomenon is also obtained that the Hamilton energy is delayed by external forcing current, which means that the neuron plays an important role in energy coding.

Keywords: Hindmarsh-Rose neuron; Hamilton energy; periodic disturbance

1. Introduction

The biological nervous system is composed of a large number of nerve cells (neurons) connected to each other and it is provided with complex dynamic behavior. To study the dynamics of neuron accurately, on the basis of electrophysiological experiment of neuron, Hodgkin and Huxley established the well-known Hodgkin-Huxley (HH) model [1] in theory and revealed electrochemical mechanism of neurophysiological activity. Thereafter, to better describe rich discharge patterns of different neurons, some noted neurophysiological models have been improved or proposed, such as FitzHugh-Nagumo (FHN) model [2], Hindmarsh-Rose (HR) model [3], Morris-Lecar (ML) model [4], Chay model [5]. Some electric activities of above mentioned neuron model or improved neuron models have been discussed and the dynamical behaviors have been presented [6-14]. For example, several resonant behaviors different from the classical deterministic oscillators were reported [6]. Song [7] investigated the dynamic behaviors influenced by noise and pointed out the response of FitzHugh-Nagumo neuron to noise. The bifurcation of the Hindmarsh-Rose neuron model in a two-dimensional parameter space was discussed [8]. A Hindmarsh-Rose neuron model with nonlinear reset process is presented and the equilibrium point or the limit cycle of the proposed system is analyzed from qualitative aspect [9]. Bifurcations of invariant sets in a five-dimensional parameter space were studied by setting appropriate system parameters in a five-dimensional parameter space [10]. Three classes of Morris-Lecar neuron to sinusoidal inputs and synaptic pulselike stimuli with deterministic and random interspike intervals were studied and it was found that two class of neurons showed similar evolutions properties, which was different from another class of neuron [11]. Parameter regions for different firing patterns in the Chay neural model were obtained by analyzing the electric activities [12]. The transitions between different electric activity modes in Chay neuronal system were explored by depolarizing current [13] and different types of bursting in it were surveyed [14].

With the development of neurodynamics, more complicated dynamical behaviors have been found in neurons or neuron networks [15-25]. For instance, diverse behaviors of time-delay HR neuron was observed with external forcing current increasing [16] and the effect of external forcing current on electric activity of neuron under magnetic flow was discussed [17]. By analyzing the pattern formation of neurons, a result is obtained that the electric activity modes could be adjusted by altering the external forcing current [21, 22]. Dynamical characteristic in an isolated neuron with memristive synapses was investigated, which confirmed that the electrical activity mode can be controlled by synapse current [23]. By analyzing numerical simulations, complex dynamical behaviors of time-delay fractional-order coupled HR neurons under electromagnetic radiation were presented [24].The dynamics of a system of two coupled Fitzhugh-Nagumo neuron system was investigated and a narrow region of parameter space of particular interest, rich with chaotic and multistable dynamics was identified[25].

Existing results [26, 27] suggest that, with the change of neuron's electrical activity, Hamilton energy neuron holding varied. This is because the energy is dependent on the discharge modes of the neuron while the mode transition of electric activities in neuron is relative to energy encoding and energy metabolism [28, 29]. By defining a Hamilton energy function, the energy shift induced by transition of electric activity mode in Hindmarsh-Rose neuron was detected [26]. Further investigation verified that the membrane potential of a neuron is dependent on the transmembrane current [30-32].

Inspired by above mentioned results, considering the electromagnetic environment in which the neurons are located. A 4D neuron model is addressed by introducing magnetic flux as a new variable into Hindmarsh-Rose neuron model. And the electric activity of the proposed model and energy dependence on the mode are discussed under periodic disturbance. Other parts of this paper are arranged as follows. In Section 2, a 4D neuron model is described and the preliminary about Hamiton energy function is given. Section 3 depicts some numerical simulations to illustrate the electric activity mode and energy dependent on the mode of the proposed neuron model. Conclusions are drawn in Section 4.

2. Model description and preliminary

In this section, introduce magnetic flux as a new variable and use linear coupling, HR neuron model [3] can be modified as

$$\begin{cases}\dot{x} = y - ax^3 + bx^2 - z + I_{ext} - \alpha x - \beta w \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x + 1.6) - z] \\ \dot{w} = x - k_1 w\end{cases}, \tag{1}$$

where x, y , z are the membrane potential, the slow current for recovery variable, and the adaption current, respectively. I_{ext} is the external forcing current. α, β, k_1 are fixed parameters describing the interaction between membrane potential x and the new variable w.

Because of wide existence of biological electricity, almost all neurons are disturbed by external forcing current. In this paper, we assume that neuron model (1) is disturbed by external forcing current

$$I_{ext} = I + Asin(\omega t + \phi), \tag{2}$$

which is a periodic signal with amplitude A, angular frequency ω and initial phase ϕ. I is a constant. Then neuron model (1) can be rewritten as

$$\begin{cases}\dot{x} = y - ax^3 + bx^2 - z - \alpha x - \beta w + I + Asin(\omega t + \phi) \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x + 1.6) - z] \\ \dot{w} = x - k_1 w\end{cases}. \tag{3}$$

As we all know, the electric activity of neuron relies on the energy release and supply, that is to say, energy storage of neuron is dependent on the external forcing and energy release is related to the electric mode. Thence, it is necessary to explore the energy transition accompanied by electric activity mode of neuron, which is induced by changing the external forcing currents. According to Helmholtz theorem [32], dynamical equations of a neuron can be regarded as a sum of conservative field and dissipative field, that is

$$f(\cdot) = f_c(\cdot) + f_d(\cdot), \tag{4}$$

where $f_c(\cdot)$ is the conservative field consisting of full rotation and $f_d(\cdot)$ is the dissipative field involving the divergence. Therefore, neuron system (3) can be broken down into

$$f_c(x,y,z,w) = J(x,y,z,w)\nabla H \begin{pmatrix} y - z - \beta w + I + A\sin(\omega t + \phi) \\ c - dx^2 \\ rs(x + K) \\ x \end{pmatrix} \tag{5a}$$

and

$$f_d(x,y,z,w) = R(x,y,z,w)\nabla H = \begin{pmatrix} -ax^3 + bx^2 - \alpha x \\ -y \\ rz \\ -k_1 w \end{pmatrix}, \tag{5b}$$