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# Numerical Approach for Bagley- Torvik Fractional Differential Equations Using Haar Wavelets

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**Abstract:** In this paper we consider Bagley-Torvik fractional differential equations, which are arising in the modeling of motion of rigid plate immersed in a Newtonian fluid. The main attribution of our content is that it transforms the fractional differential equations to a system of algebraic equations without any restrictions and assumptions. Theoretical results are authenticated by five numerical examples of both linear and nonlinear. To demonstrate the accuracy and efficiency of the Haar wavelet collocation method and results are compared with the existing methods.

**Keywords:** Fractional differential equations, Bagley-Torvik, Haar wavelets, Collocation method.

**Mathematics Subject Classifications:** 65T60, 65L60, 34A08.

## 1. Introduction

Many researchers have worked on fractional order Bagley-Torvik equations. Podlubny[1] had explained about fractional differential equations in his book. Diethelm and Ford [2] have presented the numerical solution of the Bagley-Torvik equation. Arvet and Tamme [3] have described the piecewise wise polynomial collocation method [PPCM] to solve Bagley-Torvik linear boundary value problems of fractional order. Arvet and Tamme [4, 5] have used a spline collocation method [SCM]. Jafari et al. have applied the Legendre wavelets [6], Pahdaman et al. have used the optimization technique based on training artificial neural network (ANN) to solve differential equations of fractional order [7].

The applications of fractional calculus in different fields of physics and engineering are namely dynamic of viscoelestic damped stricter [8], continuum and statistical mechanics[9], propagation of spherical flames, self similar protein dynamics[10], fluid dampers[11], bioscience, electromagnetism, signal processing control engineering[12], electrochemistry, diffusion processes, relaxation oscillation model[13], dynamics model of love, nonlinear oscillation of earthquake can be modeled with fractional derivatives[13, 14], one more important thing is a new mathematical concept to the solution of diverse problems in mathematics.

Wavelet is a mathematical tool to solve problems in science and engineering. There are many wavelets exists but Haar is the simplest wavelet in them. The graphical view of Haar scaling function is single block pulse and the mother wavelet of the Haar system is formed by two dilated unit block pulses stand by next to each other, where one of them is inverted. Haar wavelet has properties like compact support, orthogonality and simple applicability. Due to valuable properties and its simplicity Haar wavelets are using to solve problems in signal and image processing, in physics for characterization of Brownian motion, quantum field theory, numerical analysis viz. differential, integral, fractional differential equations. Over the years some researchers have worked on Haar wavelets by using different methods for numerical solutions. Lepik and Hein[15], Haariharan et al.[16], Majak et al.[17], Reddy et al.[18, 19] have given various applications of Haar wavelets with collocation method in the solution of higher order differential, integral and fractional differential equations.

**Motion of an immersed Plate:**

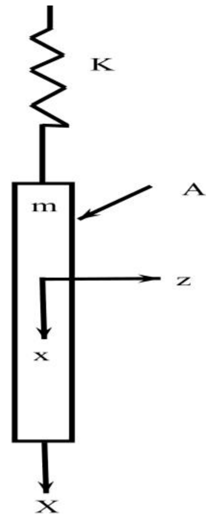


Figure. The immersed plate

Let m be the mass of the rigid plate immersed in a Newtonian fluid which is extended to infinity and connected to a fixed point by a mass less spring whose stiffness is K. Let us suppose that thefluid is not disturbed by the motion of the spring. Let A be the area of the plate which is sufficiently large in order to produce the fluid adjacent to the plate. The fluid velocity and stress are

$$\bar{v}(s,z) = \bar{v}_p(s)e^{\left(\frac{\rho s}{\mu}\right)^{0.5z}} \text{ and } \sigma(t,z) = \sqrt{\mu\rho}D_t^{0.5}[v(t,z)]. \tag{1}$$

Where, $\bar{v}_p(s)$ is the transform of the prescribed velocity of the plate. Net forces acting on the plate of displacement X, is given by

$$m\,\ddot{X} = F_x = -K\,X - 2A\,\sigma(t,0)\,. \tag{2}$$

Using equation (1) and $v_p(t,0) = \dot{X}(t)$, we obtained

$$m\frac{d^2X}{dt^2} + 2A\sqrt{\mu\rho}D^{1.5}{}_{(t)}X + K\,X = 0\,, \tag{3}$$

$$\text{Where, } D^{1.5}X = D^{0.5}\frac{dX}{dt} = \frac{d}{dt}D^{0.5}X\,. \tag{4}$$

Therefore, the presence of fractional derivative in the differential equation represents the motion of a simple physical system which includes familiar mechanical and fluid components. It may be predicted that in any systems its presence is characterized by localized motion in a viscous fluid [20].

In the present analysis we construct a simple Haar wavelet collocation method (HWCM) for the numerical solution of second order Bagley-Torvik equations of fractional order(0.5 or 1.5) initial and boundary value problems. We mainly focus on the following initial and boundary conditions carried out to confirm and certify reliability of the algorithm.

Consider the general form of Bagley-Torvik equation as follows

$$PD^2y(x) + QD^{\alpha}y(x) + R[y(x)]^k = f(x), \text{ where } \alpha = 0.5 \text{ or } 1.5,\ 0 \le x \le 1, \tag{5}$$

where, $P \neq 0,\ Q,\ R$ are constant coefficients, where k is the nonlinear integer of the equation.

Here the initial conditions given as

$$D^n y(0) = \delta_n,\ n = 0,1.$$

Whereas the boundary condition at

$$x = x_0, \text{ for } 0 < x_0 \le 1,\ D^n y(x_0) = \eta_n,\ n = 0,1.$$

$\eta_n,\ \delta_n$are real constants.