

Dynamics Analysis of a 4D Neuron model under Periodic Disturbance

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Abstract: Based on Hindmarsh-Rose neuron model, a 4D neuron model is addressed. Dynamical behaviors of the proposed neuron model are investigated under external periodic disturbance. The diversity of dynamics of the neuron is revealed via altering the parameters in the external periodic disturbance. The research results may be beneficial for further exploring the dynamic behaviors relative to the mode transition of neuron network.

Keywords: Hindmarsh-Rose neuron model; periodic disturbance; bursting

1. Introduction

To study the dynamical behaviors of neurons, a variety of simplified neuron models have been put forward for theoretical analysis and numerical research. Based on the electrophysiological experiments, Hodgkin and Huxley established the well-known Hodgkin-Huxley (HH) model considering the discharge of neuron [1], which revealed the electrophysiological mechanism of neurophysiological activity and can make people better understand the characteristics of neurons' electrical activity. On the basis of HH model, some other neuron models have been established to describe the rich discharge modes of neurons, such as FitzHugh-Nagumo (FHN) model [2], Hindmarsh-Rose (HR) model [3], Morris-Lecar (ML) model [4], Chay model [5], etc.

Some dynamical behaviors of above neurons have been discussed and presented [6-14]. For example, the bifurcation of two coupled FHN neurons was studied [6] and the dynamical properties of FHN neuron system was reproduced by adjusting the resistive-capacitive-inductance Josephson junction (RCLSJ) model [7]. The bifurcation diagram of HR neuron model in a two-dimensional parameter space was reported and the complex bifurcation structure in the diagram was pointed out [8]. Period-adding bifurcation (with or without chaos) and intermittent chaos phenomenon (periodic and intermittent chaotic) in a modified HR neuron model was observed [9]. Various responses of ML neuron under multiple stimuli were discussed [10] and the dynamics of ML neuron driven by channel noise was analyzed along with the mechanism about ion channel noise generating spontaneous action potentials being given [11]. The parameter regions for different firing patterns in Chay neural model were pointed out and the bifurcation of electric activities were analyzed [12]. The transitions between the firing activity modes in Chay neuron system were explored by depolarizing current [13] and different types of bursting in it were surveyed [14].

Among the presented neuron models, HR model is more suitable for bifurcation analysis and its output can better simulate the behaviors of some mollusk neurons. Therefore, HR neuron model has been discussed and many results about the dynamics of it have been obtained [15-18]. By discussing the pattern formation of neurons, a result is obtained that the dynamics modes could be adjusted by altering the external forcing current [19, 20].

In the existing results, the addressed neuron model, the term to describe the electro-magnetic radiation is always nonlinear, but in some real cases, the electro-magnetic radiation may be linear, which is simple and almost not being reported. Therefore, to further investigate the electric activities of neuron and reveal the dynamical behaviors of more neurons, a simplified neuron model considering magnetic flux is addressed and the dynamical behaviors of it are to be explored under periodic disturbance. Other parts of this paper will be given as follows. In Section 2, based on HR neuron model, a simplified neuron model considering magnetic flux is put forward. Section 3 depicts the simulations to illustrate the dynamical behaviors of the proposed neuron model. Conclusions are drawn in Section 4.

2. Model description

The famous HR model [3] can be described as

$$\begin{cases} \dot{x} = y - ax^3 + bx^2 - z + I_{ext} \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x + 1.6) - z] \end{cases}, \quad (1)$$

where x is the membrane potential, y is the slow current for recovery variable, and z is the adaption current. I_{ext} is the external forcing current. When system parameters of (1) are chosen as $a = 1$, $b = 3$, $c = 1$, $d = 5$, $r = 0.006$, $s = 4$ and initial values are taken as $x = -1.5$, $y = 0.7$, $z = 0.9$, the dynamical behaviors of variable x in system (1) are pictured in Fig.1 for $I_{ext} = 0.3, 1.3, 1.4, 2, 2.2, 2.7, 3, 3.5$, respectively. From Fig.1, it is easy to know that dynamics of membrane potential variable x in neuron system (1) shows diversity with I_{ext} increasing, such as quiescent state, spiking, regular bursting and chaotic bursting.

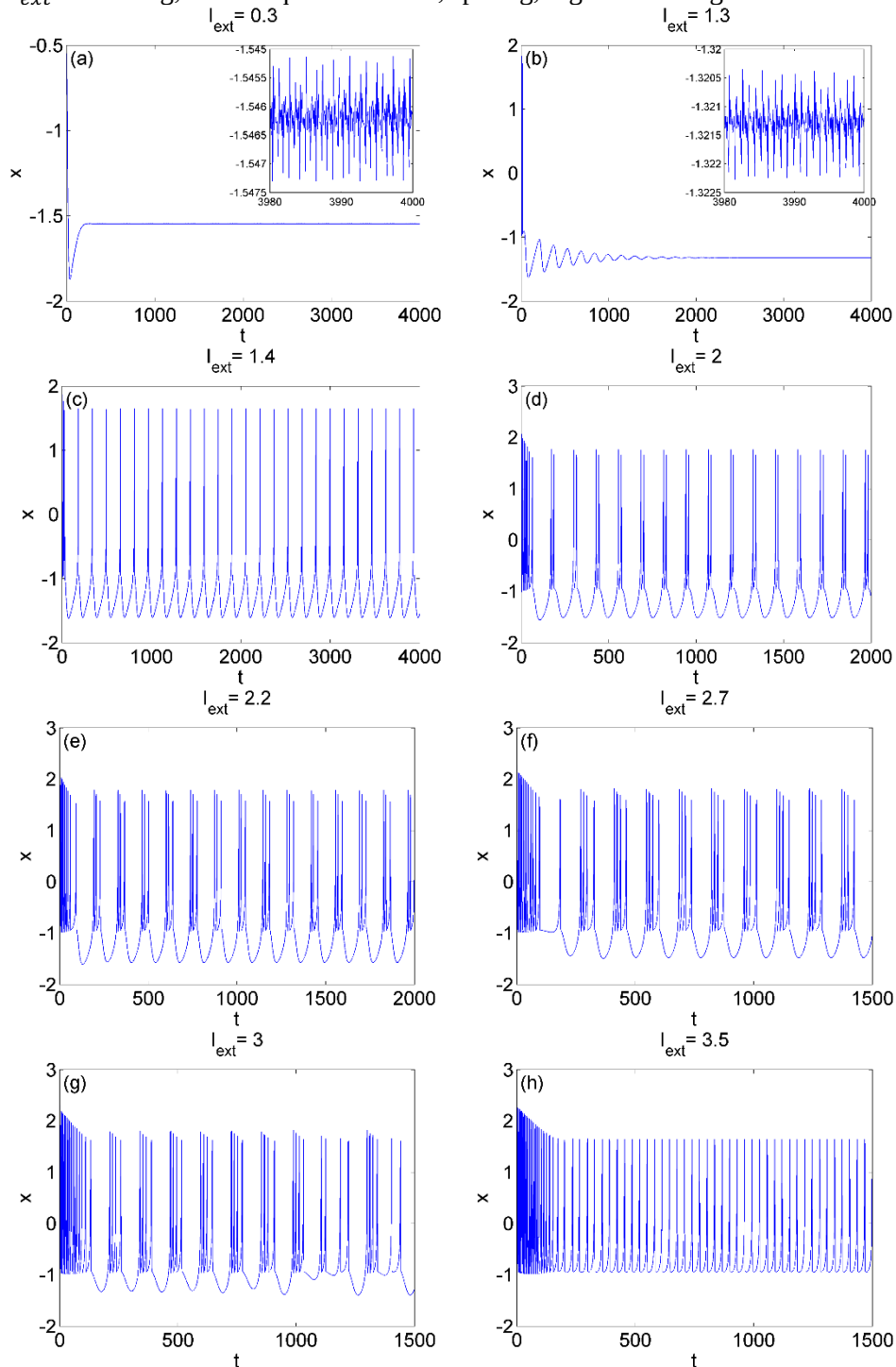


Fig.1 Dynamics of variable x in system (1) for different values of I_{ext} ,
(a) $I_{ext} = 0.3$, (b) $I_{ext} = 1.3$, (c) $I_{ext} = 1.4$, (d) $I_{ext} = 2$,