ACADEMIC World Academic Union

ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 14, No. 4, 2019, *pp*.250-258

Numerical Method for the Solution of Abel's Integral Equations using Laguerre Wavelet

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(Received March 05 2019, accepted September 28 2019)

Abstract: Laguerre wavelet based numerical method is developed for the solution of Abel's integral equations. This method is based on Laguerre wavelets basis. Laguerre wavelet method is then utilized to reduce the Abel's Integral Equations into the solution of algebraic equations. Illustrative examples are shows that the validity, efficiency and applicability of the proposed technique. This algorithm provides high accuracy and compared with other existing methods.

Keywords: Abel's integral equations; Laguerre wavelets; Collocation method.

1. Introduction

Wavelets theory is a relatively new and an emerging tool in applied mathematical research area. It has been applied in a wide range of engineering disciplines; particularly, signal analysis for waveform representation and segmentations, time-frequency analysis and fast algorithms for easy implementation. Wavelets permit the accurate representation of a variety of functions and operators. Moreover, wavelets establish a connection with fast numerical algorithms [1, 2]. Since from 1991 the various types of wavelet method have been applied for numerical solution of different kinds of integral equation, a detailed survey on these papers can be found in [3]. Such as Lepik et al. [3] applied the Haar wavelets. Maleknejad et al. proposed Legendre wavelets [4], Rationalized haar wavelet [5], Hermite Cubic splines [6], Coifman wavelet as scaling functions [7]. Yousefi et al. [8] have introduced a new CAS wavelet. Shiralashetti and Mundewadi [9] applied the Bernoulli wavelet for the numerical solution of Fredholm integral equations.

Abel's integral equation finds its applications in various fields of science and engineering. Such as microscopy, seismology, semiconductors, scattering theory, heat conduction, metallurgy, fluid flow, chemical reactions, plasma diagnostics, X-ray radiography, physical electronics, nuclear physics [10-12].

In 1823, Abel, when generalizing the tautochrone problem derived the following equation:

$$\int_0^x \frac{y(t)}{\sqrt{x-t}}dt = f(x), \qquad 0 \le x, t \le 1 \tag{1.1}$$

where f(t) is a known function and y(t) is an unknown function to be determined. This equation is a particular case of a linear Volterra integral equation of the first kind. For solving Eq. (1.1) different numerical based methods have been developed over the past few years, such as product integration methods [13, 14], collocation method [15], homotopy analysis transform method [16]. The generalized Abel's integral equations on a finite segment appeared for the first time in the paper of Zeilon [17]. Baker [18] studied the numerical treatment of integral equations. Operational matrix method based on block-pulse functions for singular integral equations [19]. Baratella and Orsi [20] applied the product integration to solve the numerical solution of weakly singular volterra integral equations. Some of the author's, have solved for Abel's integral equations using the wavelet based methods, such as Legendre wavelets [21] and Chebyshev wavelets [22]. Shahsavaran et al [23] has solved Abel's integral equation of the first kind using piecewise constant functions and Taylor expansion by collocation method. Shiralashetti [24] Theoretical study on continuous polynomial wavelet bases through wavelet series collocation method for nonlinear Lane–Emden type equations. Shiralashetti [25] applied the Laguerre wavelets collocation method for the numerical solution of the Benjamina–Bona–Mohany equations. In this paper, we introduced the Laguerre wavelets based numerical method for solving Abel's integral equations of first and second kind.

The article is organized as follows: In Section 2, the basic formulation of Laguerre wavelets and the function approximation is presented. Section 3 includes the convergence and error analysis. Section 4 is

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devoted the method of solution. In section 5, numerical results are demonstrated the accuracy of the proposed method by some of the illustrative examples. Lastly, the conclusion is given in section 6.

2. Properties of Laguerre wavelet

2.1 Wavelets

In recent years, wavelets have found their way into many different fields of science and engineering. Wavelets constitute a family of functions constructed from dialation and translation of a single function called mother wavelet. When the dialation parameter a and translation parameter b varies continuously, we have the following family of continuous wavelets:

$$\psi_{a,b}(x) = |a|^{-1/2}\psi\left(\frac{x-b}{a}\right), \forall\, a, b\ \in R, a \neq 0.$$

If we restrict the parameters a and b to discrete values as $a = a_0^{-k}, b = nb_0a_0^{-k}, a_0 > 1, b_0 > 0$. We have the following family of discrete wavelets

$$\psi_{k,n}(x) = |a|^{1/2}\psi\left(a_0^k x - nb_0\right), \forall\, a, b\ \in R, a \neq 0.$$

where $\psi_{k,n}(x)$ form a wavelet basis for $L^2(R)$. In particular, when $a_0 = 2\ and\ b_0 = 1$ then $\psi_{k,n}(x)$ forms an orthonormal basis.

2.2 Laguerre Wavelets

Laguerre wavelets $\psi_{n,m}(x) = \psi(k, n, m, x)$ have four arguments; $n = 1,2,3, \dots, 2^{k-1}$, k can assume any positive integer, m is the order of the Laguerre polynomials and x is the normalized time. They are defined on the interval [0, 1) as:

$$\psi_{n,m}(x) = \begin{cases} 2^{k/2}\bar{L}_m\left(2^k x - 2n + 1\right), \frac{n-1}{2^{k-1}} \leq x < \frac{n}{2^{k-1}}, \\ 0, \qquad otherwise \end{cases} \tag{2.1}$$

where $\overline{L_m}(x) = \frac{L_m}{m!}$, m = 0,1,2,...,M-1. In Eq. (2.1) the coefficients are used for orthonormality.
Here, $L_m(x)$ are the well-known Laguerre polynomial of order m with respect to the weight function w(x) = 1 on the interval [0, ∞) and satisfy the following recursive formula,

$$L_0(x) = 1,$$

$$L_1(x) = 1 - x,$$

$$L_{m+2}(x) = \frac{(2m + 3 - x)L_{m+1}(x) - (m + 1)L_m(x)}{m + 2}, m = 0,1,2 \dots$$

2.3 Function Approximation

A function $f(x)$ defined over [0, 1) can be expanded as a Laguerre wavelet series as follows:

$$f(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(x), \tag{2.2}$$

where, $C_{n,m}$ denotes inner product of $f(x)$ and $\psi_{n,m}(x)$

$$\text{i.e., } C_{n,m} = \big(f(x), \psi_{n,m}(x)\big). \tag{2.3}$$

If the infinite series in (2.2) is truncated, then (2.2) can be rewritten as:

$$f(x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x) = C^T\Phi(x), \tag{2.4}$$

where C and $\Phi(x)$ are $2^{k-1}M \times 1$ matrices given by:

$$C = \left[c_{10}, c_{11}, \dots, c_{1,M-1}, c_{20}, \dots, c_{2,M-1}, \dots, c_{2^{k-1},0}, \dots, c_{2^{k-1},M-1}\right]^T$$
$$= [c_1, c_2, \dots, c_{2^{k-1},M}]^T, \tag{2.5}$$

$$\Phi(x) = [\psi_{10}(x), \psi_{11}(x), \dots, \psi_{1,M-1}(x), \psi_{20}(x), \dots, \psi_{2,M-1}(x), \quad \dots, \psi_{2^{k-1},0}(x), \dots, \psi_{2^{k-1},M-1}(x)]^T$$
$$= [\psi_1(x), \psi_2(x), \dots, \psi_{2^{k-1},M}(x)]^T. \tag{2.6}$$