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The bifurcation control for a Lorenz system with time delay

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Abstract. The main aim of this manuscript is to investigate the control issue for a Lorenz system with time delay applying a hybrid control method based on state feedback and parameter perturbation. By choosing the time delay as the bifurcation parameter and utilizing stability theory for delay differential equation, the local stability in two different cases with time delay equal to 0 and not equal to 0 is discussed. With the help of Hopf bifurcation theorem, the beingness of Hopf bifurcation is established by combining the distribution results of the characteristic roots. And the hybrid control method can availably postpone the Hopf bifurcation by numerical simulations.

Keywords: Lorenz system, time delay, hybrid control, stability, Hopf bifurcation.

1. Introduction

Chaos and bifurcation control have been extensively studied in the domains of physics, mathematics, biology, and engineering in the last years. When chaos has deleterious consequences in some engineering applications, then chaos can be eliminated by control, which shows great potential in many practical use. For example, information processing, power system protection, biomedical systems, encryption and communication, etc [1-6]. The mainly explored emphasis of the bifurcation control is how to delay or eliminate the bifurcation phenomenon, in order to avoid negative consequences and purposefully establish or strengthen beneficial bifurcations for people to employ.

At present, the commonly utilized control method is feedback control method [7-9]. Cheng and Cao [7] shown the control issue of Hopf bifurcation of a delayed complex networks system. In [8], Ou et al. discussed a linear feedback controller with state variables, which is used to control the equilibrium point and periodic orbit of Lorenz system. In 2003, Luo et al. [10] firstly put forward a completely new control measure, which described by the state feedback and parameter perturbation are linked to govern the period-doubling bifurcation and chaos of discrete nonlinear systems. Liu [11] further analyzed the continuous system without time delay by utilizing a hybrid control strategy. Peng and Zhang [12-13] presented the Hopf bifurcation control of two predator-prey models by applying hybrid control method.

2. The delayed Lorenz system with control

In [14], Lian et al. proposed a Lorenz system with a time delay:

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = bx(t-\tau) - xz, \\ \dot{z} = -cz + dx^2, \end{cases} \tag{1}$$

where x, y, z are state variables, a, b, c, d are parameters of system above, τ denotes time delay, which can be understood as the hunting delay of predator to prey or delay time of signal transmission, etc. They discussed the corresponding bifurcation of system (1).

In this manuscript, in the light of the discussions above and inspired by Lian et al. [14], we design a controller which is devoted to delay the Hopf bifurcation for system (2), the corresponding mathematical model is described as:

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$$\begin{cases} \dot{x} = p[a(y-x)] + qx, \\ \dot{y} = p[bx(t-\tau) - xz] + qy, \\ \dot{z} = p[-cz + dx^2] + qz, \end{cases} \tag{2}$$

where $p > 0$ and $q \in R$ is control parameter.

This manuscript is arranged as below. In the next Section, the beingness and local stability of equilibrium point are qualitatively analyzed and the intrinsic bifurcation is postponed by discussing the relevant characteristic equation. In Section 4, we check the effectiveness and correctness of theoretical analysis by using numerical simulations. Finally, a concise conclusion is provided.

3. Existence, stability of equilibrium and Hopf bifurcation analysis

3.1. Existence of equilibrium

When $x^2 = \dfrac{abcp^3 + (ac-ab)p^2q - (a+c)pq^2 + q^3}{adp^3} \geq 0$, the system (2) has following three equilibrium: $E^*(0,0,0)$, $E^1(x, (1-\dfrac{q}{ap})x, \dfrac{pdx^2}{pc-q})$, $E^2(-x, -(1-\dfrac{q}{ap})x, \dfrac{pdx^2}{pc-q})$.

When $x^2 = \dfrac{abcp^3 + (ac-ab)p^2q - (a+c)pq^2 + q^3}{adp^3} < 0$, the system (2) has unique equilibrium $E^*(0,0,0)$.

3.2. Stability of equilibrium and Hopf bifurcation analysis

Here, we analyze the stability of system (2) at the equilibrium $E^*(0,0,0)$.

By linearizing the system (2), then we obtain

$$\begin{cases} \dot{x} = apy + (q-ap)x, \\ \dot{y} = pbx(t-\tau) + qy, \\ \dot{z} = (q-pc)z. \end{cases} \tag{3}$$

The Jacobian matrix of linearized system (3)can be written by

$$\det \begin{vmatrix} q-ap-\lambda & ap & 0 \\ pbe^{-\lambda\tau} & q-\lambda & 0 \\ 0 & 0 & q-pc-\lambda \end{vmatrix} = 0.$$

Then the characteristic equation is given by

$$\lambda^3 + m_1\lambda^2 + m_2\lambda + m_3 + (n_1\lambda + n_2)e^{-\lambda\tau} = 0, \tag{4}$$

with $m_1 = ap - 3q + cp$, $m_2 = acp^2 - 2apq - 2cpq + 3q^2$,

$m_3 = -acp^2q + apq^2 + cpq^2 - q^3$, $n_1 = -abp^2$, $n_2 = abp^2q - abcp^3$.

Next, we will discuss the local stability of the equilibrium and the conditions when Hopf bifurcation occurs. Owing to the existence of time delay in the system (2), the following two cases are considered.

Case 1: $\tau = 0$. The characteristic equation (4) becomes

$$\lambda^3 + m_1\lambda^2 + (m_2 + n_1)\lambda + m_3 + n_2 = 0. \tag{5}$$

If the condition (H1) $m_1 > 0, m_3 + n_2 > 0, m_1(m_3 + n_2) > m_2 + n_1$ is satisfied, then the total roots of Eq.(5) possess negative real parts.

Hence, when the condition (H1) holds, the equilibrium point $E^*(0,0,0)$ is locally asymptotically stable on the basis of Routh-Hurwitz criteria.

Case 2: $\tau \neq 0$.