

Semi analytical solution of MHD asymmetric flow between two porous disks

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Abstract: In this paper, we study MHD asymmetric steady incompressible viscous flow of an electrically conducting fluid between two large stationary coaxial porous disks of different permeability in the presence of uniform transverse magnetic field. The governing nonlinear momentum equations in cylindrical coordinates together with relevant boundary conditions are reduced to nonlinear ordinary differential equation (NODE) using similarity transformations. The resulting NODE is solved by Computer Extended Series Solution (CESS) and Homotopy Analysis Method (HAM). The analytical solutions are explicitly expressed by recurrence relation for determining the universal coefficients. The nearest singularity is obtained at $R=4.2981$ with help of Domb-Sykes plot which restricts the convergence of the series, using Euler transformation the singularity is mapped to infinity. The obtained solutions are valid for all values of the Reynolds number, magnetic parameter and permeability parameter are presented through graphs and tabular forms to discuss the important features of the flow. The resulting solutions are compared with the earlier literatures which are found to be in good agreement. Further, the region of validity of the series is extended for much larger values of R up to infinity by Pade' approximants.

Keywords: MHD; asymmetric flow; CESS; Domb-Sykes plot; Euler transformation; HAM and Pade' approximants.¹

1. Introduction

The analysis of MHD viscous fluid flow between two parallel porous disks is an interesting area of research because of its significant applications in many fields of science and engineering such as hydrodynamical machines and apparatus, magnetic storage devices, thermal power generating systems, medical equipments, rotating machinery, computer storage devices, gas turbine engines, air cleaning machines, crystal growth processes, geothermal, oceanography, geophysical, biomechanics, design of thrust bearings and most importantly aerodynamic applications etc. Siddique *et al.* [1] and Khan *et al.* [2] investigated the flow of a second grade fluid for MHD transient rotation flow and MHD flow between two side walls perpendicular to a plate in a porous medium respectively. Hayat *et al.* [3] discussed the effects of radiation on MHD flow of a Maxwell fluid analytically. Fetecau *et al.* [4] investigated exact solutions for the helical flow of an Oldroyd-B fluid in a circular cylinder. Ashraf *et al.* [5] studied numerically for the problem of MHD stagnation point flow through a micropolar fluid towards a heated surface. Rudraiah and Chandrasekhara [6] have discussed inner and outer solutions for the MHD flow between parallel porous disks for large suction Reynolds. Rasmussen [7] studied numerically the problem of steady viscous flow between two parallel porous disks. Elcrat [8] discussed the existence and uniqueness of the problem on radial flow of viscous fluid between two coaxial permeable disks. Guar and Chaudhary [9] investigated the effect of heat transfer for laminar flow between parallel disks of different permeability. Phan-Thien and Bush [10] presented an exact solution for the problem of steady viscous flow of a Newtonian fluid between two porous disks. Rajagopal *et al.* [11-12] discussed the asymmetric flow above a rotating disk and also obtained the numerical solution for the asymmetric flow between parallel rotating disks for a set of values of the governing parameters. Singh *et al.* [13] investigated the experimental and numerical results to discuss the effect of acceleration on flow field. Attia [14] analyzed the effectiveness of the ion slip on the steady flow of an incompressible viscous electrically conducting fluid due to a porous rotating disk with heat transfer. Ersoy [15] examined an approximate solution for the viscous fluid flow between disks rotating about distinct vertical axes for different speeds to study the dependence of velocity fields on the position, the Reynolds number, the eccentricity, and the ratio of angular speeds of the disks. Fang and Zhang [16]

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find an exact solution for the problem of an axisymmetric flow over a stretchable disk by neglecting the body forces and also discussed the effects of disk stretching and stretching Reynolds number with the help of Von Karman's similarity transformations. Bhatt and Hamza [17] presented the similarity solution for the squeezed film flow between two rotating naturally permeable disks.

In this paper, we present the series solutions for the MHD asymmetric flow between porous disks of different permeability in the presence of transverse magnetic field. We obtain the solution for small as well as large Reynolds number using CESS and HAM where as previous authors [18] presented the solutions based on finite difference method for small Reynolds number. In the first method, we investigate the flow problem using CESS based on regular perturbation method. In CESS method, we first calculate sufficiently large number terms of the low Reynolds number perturbation series by using Mathematica so that the nature and location of the singularity can be predicted quite accurately by using Domb-Sykes plot. Secondly, the analytic continuation is used effectively for extending the validity of the perturbation series to moderately higher Reynolds number. Euler transformation is used for the analysis and improvement of the series. Further, we use Pade approximants for summing the Eulerised series to extend the validity of the series for large value of Reynolds number up to infinity. The salient features of this method are evidently explained by Van Dyke [19]. Bujurke and his associates [20-23] have clearly shown the potential applications of these methods in computational fluid dynamics.

We also investigate the same flow problem using fast converging semi-analytical method called Homotopy analysis method (HAM) proposed by Liao [24]. The HAM provides us with a convenient way to control and adjust the convergence region and rate of approximating the solution by series. In this method we have the liberty to choose base functions of the required solution and the corresponding auxiliary linear operator. Therefore, the HAM has great features and advantages over all other analytical approximate methods, so it is also easy to use for the physical problems arises in flow problems. Recently, Awati et al. [25] studied the solution of MHD flow of viscous fluid between parallel porous plates in the presence of the magnetic field using CESS and HAM.

The paper is outlined as follows. Section 1 is devoted to describe the introduction; section 2 develops the mathematical formulation of the proposed problem with relevant boundary conditions. The solution of the problem is obtained by Computer extended series and Homotopy analysis method in section 3 and 4 respectively. Section 5 presents results and discussion; section 6 is about the conclusion.

2. Mathematical Formulation

Let us consider the MHD asymmetric steady incompressible viscous flow of an electrically conducting fluid between two stationary coaxial porous disks of infinite radii coinciding with the planes $z = \pm a$ with constant injection velocities V_1 and V_2 at the lower and upper disks respectively in the presence of a uniform transverse magnetic field of intensity B_0 . The geometry of the governing physical flow problem is shown in Fig. 1.

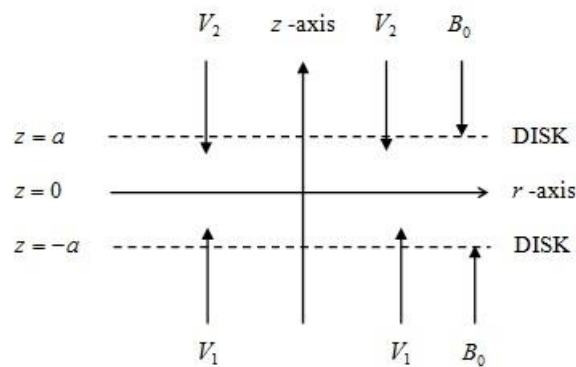


Fig.1 Geometry of the physical phenomenon.

In order to investigate the effects of different permeability of the disks it's necessary to define the following permeability parameter as follows

$$A = 1 - \frac{V_1}{V_2} \quad (2.1)$$