ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 15, No. 1, 2020, *pp*.052-066

Bezier Polynomials and its Applications with the Tenth and Twelfth Order Boundary Value Problems

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(Received April 12, 2020, accepted May 30, 2020)

Abstract: The aim of this paper is to apply Galerkin weighted residual method for solving tenth and twelfth order linear and nonlinear boundary value problems (BVPs). A trial function is assumed which is made to satisfy the boundary conditions given, and used to generate the residual to be minimized. The method is formulated as a rigorous matrix form. To investigate the effectiveness of the method, numerical examples were considered which were compared with both the analytic solutions and the solutions obtained by our method. It is observed that, the proposed method is very accurate, better, efficient and appropriate. All problems are computed using the software MATLAB.

Keywords: Numerical solutions, Linear and nonlinear tenth and twelfth order BVPs, Galerkin method, Bezier polynomials.

I. Introduction

Tenth and Twelfth order boundary value problems arise in the study of fluid dynamics, hydro magnetic stability, beam and long wave theory, physics, engineering and applied sciences. Owing to their mathematical significance and applications, several methods such as finite difference method, decomposition method and polynomial spline have been used to solve these types of problems. From the literature we observe that, Siddiqi and Twizell [1] solved tenth order BVPs using tenth degree spline where some unforeseen results for the solution and higher order derivatives were acquired near the boundaries of the interval. Siddiqi and Ghazala [2] acquainted the solutions of tenth order BVPs by eleventh degree spline. A reliable algorithm for solving tenth order BVPs using variational iterative method is developed by Muhammad Aslam Noor et al [3]. On the other hand, variational iteration method for the numerical solution of tenth orders BVPs were used by Fazhan and Xiuying [4]. Inayat Ullah et al [5] acquainted the numerical solutions of higher order nonlinear BVPs by new iterative method. Siddiqi and Twizell [6, 7] solved the tenth and twelfth order BVPs using tenth and twelfth degree splines respectively. Siddiqi and Ghazala Akram [8, 9] elaborated the solutions of tenth and twelfth order BVPs applying eleventh and thirteen degree spline respectively. Approximate solutions of twelfth order BVPs were acquainted by Mohy-ud-Din et al [10]. Mirmoradi et al [11] used Homotopy perturbation method to solve Tenth order and Twelfth order boundary value problems.
This article is organized as, in section II, basic concept of Bezier polynomials are introduced. In section III, two formulations for solving linear and nonlinear higher order BVPs including two types of boundary conditions by Galerkin residual method are presented. The proposed formulation is verified on three linear and two nonlinear BVPs in section IV. Finally, in the last section, the conclusion of the paper is inserted.

II. Bezier Polynomials

The Bezier polynomials of nth degree form a complete basis over [0, 1] and they are defined by

$$B_{j,n}(x) = \sum_{j=0}^{n} \binom{n}{j} x^j (1-x)^{n-j} P_j \, , 0 \le x \le 1$$

Where the binomial coefficients are given by

$$\binom{n}{j} = \frac{n!}{(n-j)!\, j!}$$

The points P_j are called control points for the Bezier curve.
We write first 20 Bezier polynomials of degree 19 over the interval [0,1]:

$B_0(x) = (1-x)^{19}$
$B_1(x) = 19(1-x)^{18}x$
$B_2(x) = 171(1-x)^{17}x^2$
$B_3(x) = 969(1-x)^{16}x^3$
$B_4(x) = 3876(1-x)^{15}x^4$
$B_5(x) = 11628(1-x)^{14}x^5$
$B_6(x) = 27132(1-x)^{13}x^6$
$B_7(x) = 50388(1-x)^{12}x^7$
$B_8(x) = 75582(1-x)^{11}x^8$
$B_9(x) = 92378(1-x)^{10}x^9$
$B_{10}(x) = 92378(1-x)^9x^{10}$
$B_{11}(x) = 3876(1-x)^8x^{11}$
$B_{12}(x) = 75582(1-x)^7x^{12}$
$B_{13}(x) = 27132(1-x)^6x^{13}$
$B_{14}(x) = 11628(1-x)^5x^{14}$
$B_{15}(x) = 3876(1-x)^4x^{15}$
$B_{16}(x) = 969(1-x)^3x^{16}$
$B_{17}(x) = 171(1-x)^2x^{17}$
$B_{18}(x) = 19(1-x)x^{18}$
$B_{19}(x) = x^{19}$

Since Bezier polynomials have special properties at $x = 0$ and $x = 1 : B_{j,n}(0) = 0$ and $B_{j,n}(1) = 0, j = 1,2,\dots,n-1$ respectively, so that they can be used as set of basis function to satisfy the corresponding homogeneous form of the essential boundary conditions to derive the matrix formulation in the Galerkin method to solve a BVP over the interval [0,1].

III. Formulation of BVPs in Matrix Form

In this section, we first obtain the rigorous matrix formulation for tenth order linear BVP and then we extend our idea for solving nonlinear BVP. For this, we consider a linear tenth order differential equation given by

$$a_{10}\frac{d^{10}u}{dx^{10}} + a_9\frac{d^9u}{dx^9} + a_8\frac{d^8u}{dx^8} + a_7\frac{d^7u}{dx^7} + a_6\frac{d^6u}{dx^6} + a_5\frac{d^5u}{dx^5} + a_4\frac{d^4u}{dx^4} + a_3\frac{d^3u}{dx^3} + a_2\frac{d^2u}{dx^2} + a_1\frac{du}{dx} + a_0u = r,\ a < x < b \quad (1a)$$

subject to the following two types of boundary conditions:

Type 1

$$u(a) = A_0,\ u(b) = B_0,\ u'(a) = A_1,\ u'(b) = B_1,\ u''(a) = A_2,\ u''(b) = B_2,\ u'''(a) = A_3,$$
$$u'''(b) = B_3,\ u^{(iv)}(a) = A_4,\ u^{(iv)}(b) = B_4 \quad (1b)$$

Type 2

$$u(a) = A_0,\ u(b) = B_0,\ u''(a) = A_2,\ u''(b) = B_2,\ u^{(iv)}(a) = A_4,\ u^{(iv)}(b) = B_4,\ u^{(vi)}(a) = A_6,\ u^{(vi)}(b) = B_6,$$
$$u^{(viii)}(a) = A_8,\ u^{(viii)}(b) = B_8 \quad (1c)$$

Where $A_i, B_i, i = 0,1,2,3,4,5,6,8$ are finite real constants and $a_i, i = 0,1,2,3,4,5,6,7,8,9,10$ and r are all continuous and differentiable functions of x defined on the interval [a, b]. The BVP (1) is solved with both the boundary conditions of type 1 and type 2.

Since we want to use the polynomials, described in section II, as trial functions which are derived over the interval [0, 1], so the BVP (1) is to be converted to an equivalent problem on [0, 1] by replacing x by $(b-a)x + a$, and thus we have:

$$c_{10}\frac{d^{10}u}{dx^{10}} + c_9\frac{d^9u}{dx^9} + c_8\frac{d^8u}{dx^8} + c_7\frac{d^7u}{dx^7} + c_6\frac{d^6u}{dx^6} + c_5\frac{d^5u}{dx^5} + c_4\frac{d^4u}{dx^4} + c_3\frac{d^3u}{dx^3} + c_2\frac{d^2u}{dx^2} + c_1\frac{du}{dx} + c_0u = t,\ 0 < x < 1 \quad (2a)$$

$$u(0) = A_0,\ u(1) = B_0,\ \frac{1}{(b-a)}u'(0) = A_1,\ \frac{1}{(b-a)}u'(1) = B_1,\ \frac{1}{(b-a)^2}u''(0) = A_2,$$
$$\frac{1}{(b-a)^2}u''(1) = B_2,\ \frac{1}{(b-a)^3}u'''(0) = A_3,\ \frac{1}{(b-a)^3}u'''(1) = B_3,\ \frac{1}{(b-a)^4}u^{(iv)}(0) = A_4,\ \frac{1}{(b-a)^4}u^{(iv)}(1) = B_4 \quad (2b)$$

and

$$u(0) = A_0,\ u(1) = B_0,\ \frac{1}{(b-a)^2}u''(0) = A_1,\ \frac{1}{(b-a)^2}u''(1) = B_1,\ \frac{1}{(b-a)^4}u^{(iv)}(0) = A_2,$$
$$\frac{1}{(b-a)^4}u^{(iv)}(1) = B_2,\ \frac{1}{(b-a)^6}u^{(vi)}(0) = A_3,\ \frac{1}{(b-a)^6}u^{(vi)}(1) = B_3,\ \frac{1}{(b-a)^8}u^{(viii)}(0) = A_4,\ \frac{1}{(b-a)^8}u^{(viii)}(1) = B_4 \quad (2c)$$

where