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One input control and synchronization for generalized Lorenz-like systems

Yawen Wu and Shunjie Li[1]
Nanjing University of Information Science and Technology, Nanjing211800, China
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Abstract. This paper proposes a new class of nonlinear systems called generalized Lorenz-like systems which can be used to describe many usual three-dimensional chaotic systems such as Lorenz system, Lü system, Chen system, Liu system, etc. Then the control and synchronization problems for generalized Lorenz-like system via a single input are studied and two control laws are proposed based on partial feedback linearization with asymptotically stable zero dynamics. Finally, the numerical simulations demonstrate the correctness and effectiveness of the proposed control strategies.

Keywords: Chaos synchronization; zero dynamics; generalized Lorenz-like system.

1. Introduction

In the past three decades, the topic of control and synchronization for chaotic systems has attracted increasing attentions because of its possible applications in secure communication [1-2], biomedical Engineering [3] and etc. The chaos synchronization was introduced, in 1990, by Pcora and Carroll [4], which is used to synchronize two identical chaotic systems with different initial conditions. Since then, a wide variety of methods of the control and synchronization for chaotic systems have been proposed, such as linear feedback control method [5-6], sliding mode control [7], adaptive control method [8-9], backstepping control method [10-11] and so on.

It is well known that if a nonlinear control system is partial feedback linearizable and its corresponding zero dynamics is asymptotically stable, then the control that stabilizes the linear sub-system will stabilize the original system [12-15]. In this paper, a class of generalized Lorenz-like system is introduced which can describe many usual chaotic systems such as Lorenz system, Chen system, Liu system, Lü system and etc. Our object is to realize the control and synchronization, for any given initial conditions, of generalized Lorenz-like system by one input. Two one-input control strategies are proposed for the control and synchronization, respectively, based on partial feedback linearization with asymptotically stable zero dynamics of the corresponding error systems.

This paper is organized as follows. In Section 2, the generalized Lorenz-like system is introduced and moreover, useful notations and problem statement is also given. The main results are presented in Section 3. Numerical simulations are shown in Section 4 to verify the effectiveness and correctness of the proposed one-input control strategies. Finally, concluding remarks are given in Section 5.

2. Preliminaries and problem statement

2.1 Zero dynamics [12-13]

Consider a single-input single-output nonlinear system

$$\Sigma: \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases}$$

where the state $x \in \mathbb{R}^n$, the control $u \in \mathbb{R}$ and the entries f, g are smooth vector fields on $\mathbb{R}^n$. Let$y = h(x)$ be an output of Σ with relative degree $r < n$ at some point x_0, then locally there exist a regular static state feedback $u = \alpha(x) + \beta(x)v$ and a state transformation $z = (z^1, z^2) = (\Phi^1(x), \Phi^2(x)) = \Phi(x)$, where $z^1 = (z_1, \dots, z_r)^\top, z^2 = (z_{r+1}, \dots, z_n)^\top$, and Φ is a diffeomorphism, such that in the $z-$coordinates, the system Σ reads, locally,

[1] Corresponding author. E-mail address: shunjie.li@nuist.edu.cn

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
&\vdots \\
\dot{z}_{r-1} &= z_r \\
\dot{z}_r &= v \\
\dot{z}^2 &= \eta(z^1, z^2) \\
y &= z_1
\end{aligned}
$$

Definition 1. The zero dynamics of system Σ is defined by the dynamics $\dot{z}^2 = \eta(0, z^2)$ which are the internal dynamics consistent with the constraint that $y(t) \equiv 0$.

Lemma 2. If the zero dynamics of system Σ is asymptotically stable, then the control u that stabilizes the linear sub-system will stabilize the system Σ.

2.2 Generalized Lorenz system

Consider a nonlinear autonomous system defined on $\mathbb{R}^3$

$$\Lambda: \ \dot{x} = Ax + f(x),$$

where the state $x = (x_1, x_2, x_3)^T$, the smooth vector field $f(x) = (f_1(x), f_2(x), f_3(x))^T$, is the quadratic nonlinear part of system and A is a constant matrix which is in the following form:

$$A = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}.$$

Definition 3. The nonlinear system is called a generalized Lorenz-like system if it satisfies $a_{33} < 0, a_{12} \neq 0$ and $f_1(x) = 0,\ f_2(x) = f_2(x_1, x_3),\ f_3(x) = f_3(x_1, x_2)$. In other words, the generalized Lorenz-like system is in the following form

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + f_2(x_1, x_3) \\ \dot{x}_3 = a_{33}x_3 + f_3(x_1, x_2). \end{cases} \tag{1}$$

Remark 4. Many usual chaotic systems can be described by the generalized Lorenz-like system. For example, when $a_{11} < 0, a_{12} = -a_{11}, f_2(x_1, x_3) = lx_1x_3$ and $f_3(x_1, x_2) = hx_2^2$, it becomes Multi-wing system [16]. Moreover, it is easy to see that Lorenz system [17], Chen system [2], Liu system [18], Lü system [19], etc., can also be described by this class.

Remark 5. In [20], a similar nonlinear control system called generalized Lorenz system was introduced in which the elements of $f(x)$ were defined by $f_1(x) = 0, f_2(x_1, x_3) = x_1x_3, f_3(x_1, x_2) = -x_1x_2$. Therefore, the system (1) is more generalized than that in [20].

2.3 Problem statement

In this paper, the control and synchronization by one input for the generalized Lorenz-like system is studied and the control strategies are proposed based on the partial feedback linearization with asymptotically stable zero dynamics. More precisely, we add a control variable to the second equation of (1),

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + f_2(x_1, x_3) + u \\ \dot{x}_3 = a_{33}x_3 + f_3(x_1, x_2). \end{cases} \tag{2}$$

which is called the slave system with $a_{33} < 0, a_{12} \neq 0$ and the master system denotes the original system in variable y:

$$\begin{cases} \dot{y}_1 = a_{11}y_1 + a_{12}y_2 \\ \dot{y}_2 = a_{21}y_1 + a_{22}y_2 + f_2(y_1, y_3) \\ \dot{y}_3 = a_{33}y_3 + f_3(y_1, y_2). \end{cases} \tag{3}$$

The object of this paper is to solve the following control and synchronization problems for generalized Lorenz-like system (1) via single input:

(i) For any equilibrium point (x_1^*, x_2^*, x_3^*), find a suitable control u such that $\lim\limits_{t\to\infty}|x - x^*| = 0$ for any initial condition $(x_1(0), x_2(0), x_3(0))$;

(ii) Find a suitable control u such that $\lim\limits_{t\to\infty}|x - y| = 0$ for any initial conditions $(x_1(0), x_2(0), x_3(0))$ and $(y_1(0), y_2(0), y_3(0))$.