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Galerkin method for the numerical solution of boundary value problems using Boubaker wavelets

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Abstract. In this paper, we proposed Galerkin method for the numerical solution of boundary value problems using Boubaker wavelets. Here, we used weight functions as Boubaker wavelets that are assumed bases elements which allow us to obtain the numerical solutions of the boundary value problems. Obtained numerical results using this method are compared with the existing method and exact solution. Some test problems are considered to demonstrate the validity and applicability of the proposed method.

Keywords: Galerkin method; Boubaker wavelets; Boundary value problems..

1. Introduction

Boundary value problems (BVPs) occur frequently in the fields of engineering and science such as gas dynamics, nuclear physics, atomic structures and chemical reactions. In most cases, we do not always find the exact solutions for these equations via analytical methods. In this case, it is very meaningful to give the high precision numerical solutions for this kind of problem by numerical methods. Recently, some of the numerical methods are used for the numerical solutions of such type of problems. For example, Haar wavelet collocation method [1], Legendre wavelet collocation method [2], Series solution [3] etc.

The subject of wavelets has received much attention because of the comprehensive mathematical power and the good application potential of wavelets in many interesting physical problems. Wavelet functions have generated significant interest from both theoretical and applied research over the last few years. The name wavelet comes from the requirement that they should integrate to zero, waving above and below x-axis. The concepts for understanding wavelets were provided recently by Meyer, Mallat, Daubechies, and many others. Since then, the number of applications where wavelets have been used has exploded. Many different types of wavelet functions have been presented over the past few years [4].

In wavelet theory, the contribution of orthogonal bases of compactly supported wavelet by Daubechies (1988) and Multiresolution analysis based fast wavelet transform algorithm by Belkin (1991), wavelet based approximation of ordinary differential equations gained momentum in attractive way. Special interest has been dedicated to the construction of compactly supported smooth wavelet bases. As we have noted earlier that, spectral bases are infinitely differentiable but have global support. On the other side, bases functions used in finite-element methods have small compact support but poor continuity properties. Already we know that, spectral methods have good spectral localization but poor spatial localization, while finite element methods have good spatial localization, but poor spectral localization. Wavelet bases execute to combine the advantages of both spectral and finite element bases. We can look forward to numerical methods based on wavelet bases to be capable to attain good spatial and spectral resolutions. Representation of a smooth function in terms of a series expansion using orthogonal polynomials is a fundamental idea in approximation theory and forms the basis of spectral methods of solution of differential equations with functional arguments. An approach to study differential equations is the use of wavelet function bases in place of other conventional piecewise polynomial trial functions in finite element type methods. Because of its implementation simplicity, the Galerkin method is considered the most widely used in applied mathematics [5-6].

The advantage of wavelet-Galerkin method over finite difference or finite element method has lead to tremendous applications in science and engineering. To a certain extent, the wavelet technique is a strong competitor to the finite element method. Also, the wavelet method provided an efficient alternative technique for solving boundary value problems numerically.

In this paper, we developed Galerkin method for the numerical solution of boundary value problems using Boubaker wavelets. This method is based on expanding the solution by Boubaker wavelets with unknown coefficients. The properties of Boubaker wavelets together with the Galerkin method are utilized to evaluate the unknown coefficients and then a numerical solution of the boundary value problems is obtained.

The organization of the paper is as follows. Preliminaries of Boubaker wavelets and function approximation are given section 2. Section 3 deals with Boubaker wavelet-Galerkin method for the solution of boundary value problems. Numerical Experiment is given in section 4. Finally, conclusions of the proposed work are discussed in section 5

2. Boubaker wavelets and Function approximation

Boubaker wavelets:

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b varies continuously, we have the following family of continuous wavelets [7]:

$$\psi_{a,b}(x) = |a|^{\frac{-1}{2}}\psi\left(\frac{x-b}{a}\right) \rightleftarrows\rightleftarrows, a, b \in R \& a \neq 0$$

If we restrict the parameters $a\&b$ to discrete values as

$$a = a_0^{-n}, b = mb_0a_0^{-n}; a_0 > 1, b_0 > 0$$

we have the following family of discrete wavelets

$$\psi_{n,m}(x) = |a_0|^{\frac{1}{2}}\psi(a_0^n x - mb_0), n, m \in Z$$

Where $\psi_{n,m}$ form a wavelet basis for a, b. In particular, when $a_0 = 2\&b_0 = 1$, then $\psi_{n,m}(x)$ forms an orthonormal basis.
Boubaker wavelets are defined as follows:

$$\psi_{n,m}(x) = \begin{cases} \sqrt{2m+1}\frac{(2m!)}{(m!)^2}2^{\frac{k+1}{2}}B_m\left(2^{k+1}x - 2n + 1\right), \frac{n-1}{2^{k-1}} \leq x < \frac{n}{2^{k-1}} \\ 0, Otherwise \end{cases} \quad (2.1)$$

where, k is any positive integer, $n = 1,2,3,\ldots\ldots\ldots, 2^{k-1}$ is an argument and $m = 0,1,2,3.\ldots\ldots.M-1$ is the order of Boubaker functions

$B_0(x) = 1$, $B_1(x) = \frac{1}{2}(2x-1)$.

$B_2(x) = \frac{1}{6}(6x^2 - 6x + 1)$, $B_3(x) = \frac{1}{20}(20x^3 - 30x^2 + 12x - 1)$

and so on. For instance, for $k = 1$ and $M = 4$, we get the Boubaker wavelet bases as follows:

$\psi_{1,0}(x) = 2$,

$\psi_{1,1}(x) = 2\sqrt{3}(8x-3)$,

$\psi_{1,2}(x) = 2\sqrt{5}(96x^2 - 72x + 13)$,

$\psi_{1,3}(x) = 2\sqrt{7}(1280x^3 - 1440x^2 + 528x - 63)$ and so on.

Function approximation:

Suppose $y(x) \in L^2 0,1)$ is expanded in terms of Boubaker wavelets as:

$$y(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\,\psi_{n,m}(x) \quad (2.2)$$

Truncating the above infinite series, we get

$$y(x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M} c_{n,m}\,\psi_{n,m}(x) \quad (2.3)$$

3. Method of solution

Consider the boundary value of the problem is of the form,

$$y'' + \alpha y' + \beta y = f(x) \quad (3.1)$$