

# Dynamics Analysis of a Kind of Chaotic System under Periodic Excitation

Lizhou Zhuang<sup>1</sup>, Xuerong Shi<sup>2</sup>, Zuolei Wang<sup>2\*</sup>

<sup>1</sup> School of Chemical and Environmental Engineering, Yancheng Teachers University,  
Yancheng 224002, China

<sup>2</sup> School of Mathematics and Statistics, Yancheng Teachers University, Yancheng 224002, China  
(Received April 21, 2021, accepted September 12, 2021)

**Abstract:** The dynamical behaviors of the novel four-dimensional memristor self-oscillated system with periodic excitation are discussed. With the change of amplitude and angular frequency of external periodic excitation, the proposed system can generate various dynamics including chaotic, periodic and bursting oscillations. Using the numerical simulation method, the phase trajectory and time series are employed to verify the behavior of the considered system.

**Keywords:** Chaos system, Periodic excitation, Dynamical behavior

## 1. Introduction

Distinct from classical attractors, such as Lorenz attractor [1], Chen attractor [2], Lü attractor [3], and others [4-6], hidden attractor is a new type of attractor [7-11]. It was found that some chaotic systems without equilibrium or stable equilibrium can appear chaotic attractor. The hidden attractor has attracted much attention [12-15]. Additionally, the hidden attractor is often unwanted in many cases of practical application. To avoid or utilize the hidden attractor, it is meaningful and important to investigate the characteristic of it [16-21].

In recent years, a variety of chaotic systems with memristor have attracted much attention [22-26]. Bao et al. presented a new type of four-dimensional self-oscillated system with no equilibrium [22]. The coexistence of various hidden attractors including periodic solution, quasi-periodic periodic solution, chaotic attractors was studied. The Chaotic system could also display abundant dynamical behaviors under external stimulation. Wang et al. described a time-delay memristive Hindmarsh-Rose neuron model, and the transition of electrical activities of the neuron was investigated with the change of noise intensity [27]. Li et al. presented a Duffing system with periodic excitation and theoretically analyzed the dynamic phenomena of the system under different parameters [28]. For chaotic system with periodic excitation, abundant dynamical behaviors were investigated because of extra periodic excitation [29-31]. It is useful to explore the dynamics under periodic excitation. In this paper, variety of dynamical behaviors of the novel four-dimensional memristor self-oscillated system is discussed under external periodic excitation. With the change of amplitude and angular frequency of periodic excitation, the dynamics of proposed system is discussed, including chaotic, periodic, and bursting oscillations as well as coexistence of different dynamics. Other parts of this paper are arranged as follows. Section 2 presents a chaotic system with periodic excitation. The various dynamics of system are revealed in Sections 3. Conclusion are given in Section 4.

## 2. System description

A four-dimensional chaotic system is presented in Ref.1, including two linearly coupled term, one constant term and four nonlinear terms. The equations are described as

$$\begin{cases} \dot{x} = y \\ \dot{y} = (z + x^2 - \beta x^4)y - \omega_0^2 W(w)x, \\ \dot{z} = \mu - x^2 \\ \dot{w} = x - w \end{cases} \quad (1)$$

where  $\beta$ ,  $\mu$  and  $\omega_0$  are parameters controlling the dynamics of system (1),  $W(w) = a + b|w|$  is non-ideal voltage-controlled memristor with  $a, b$  being intrinsic parameter [22]. To investigate the dynamics of system (1) under periodic excitation, system (1) can be revised as

$$\begin{cases} \dot{x} = y + A \sin(\omega t) \\ \dot{y} = (z + x^2 - \beta x^4)y - \omega_0^2 W(w)x, \\ \dot{z} = \mu - x^2 \\ \dot{w} = x - w \end{cases}, \quad (2)$$

where  $A$  and  $\omega$  are the amplitude and angular frequency of periodic excitation, respectively. It is obvious that system (2) is a nonautonomous system.

### 3. Dynamics analysis of the proposed system

With appropriate parameters, system (1) can exhibit chaotic attractors or periodic solution. In this section, the dynamics of system (2), namely, system (1) under periodic excitation, is to be explored with the change of amplitude and angular frequency of periodic excitation. For this end, we suppose that  $\omega_0$  and  $A$  change, respectively, while other parameters are fixed as  $\beta=0.5$ ,  $\mu=0.9$ ,  $a=1$ ,  $b=0.1$ ,  $\omega = \omega_0$ .

As the parameter  $\omega_0$  is chosen as 1.68, chaotic attractor of system (1) is generated under initial values  $(-2, 0, 0, 0)$  [22]. The phase portraits in  $x - y$  plane is displayed in Fig.1.

Keeping other parameters and initial condition constant, the dynamic of system (2) is calculated via choosing different value of stimulus amplitude and depicted in Fig.2. From Fig.2, it can be obtained that period-3 behavior is presented when  $A = 0.4$ ; when  $A = 0.6$ , the period-1 behavior will appear. Thus, the system presents different dynamics as the amplitude changing. To better illustrate this, we also discuss other scenarios.

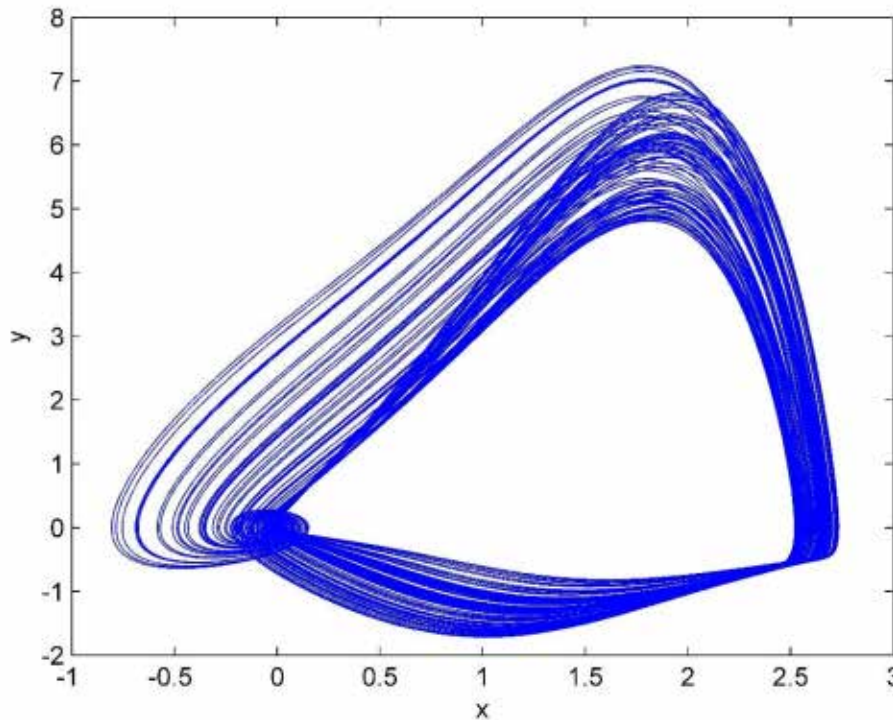


Fig.1. Phase portrait of system (1) in  $x - y$  plane with initial condition  $(-2, 0, 0, 0)$  and  $\omega_0=1.68$ .