DOI: 10.4208/JICS-2023-006 December 2023

## **Applications of Multifractal Detrended Fluctuation Analysis**

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**Abstract.** In recent years, multifractal detrended fluctuation analysis (MF-DFA) has become an important tool for detecting the scale and long correlation of non-stationary time series. With the continuous development of multifractal theory, researchers have widely applied it in physics, chemistry, biology, economy, etc. In this paper, we briefly review various applications of MF-DFA, and present some empirical research using one- and two-dimensional (2D) MF-DFA. 1D MF-DFA is always applied in financial markets, energy markets, heartbeat, and atmospheric science. Furthermore, 2D MF-DFA has been studied in surface science such as image segmentation, medical image classification. In this paper, we use 1D MF-DFA to explore the market efficiency of Korean stock market, and adopt 2D MF-DFA to segment images such as the license plate and hepatic cell image. In addition, we apply the proposed algorithm to segment transmission lines under icing condition, and the proposed method achieves satisfactory segmentation results.

AMS subject classifications: 28A80, 94A08

**Key words**: MF-DFA, Hurst exponent, Feature extraction, Image segmentation.

## 1 Introduction

Fractal phenomena are common in the natural and social sciences. Since Kantelhardt et al. [1] proposed the MF-DFA in 2002, this method has been successfully applied in various fields. The advantage of this method is that it can systematically filter out the trend components of each order, and can detect the correlation of the signal with noise.

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In the past decades, one dimensional multifractal detrended fluctuation analysis (1D MF-DFA) has been widely used in financial markets such as stock markets [2], exchange markets [3], crude oil markets [4], and Bitcoin market [5]. Matia et al. [6] used this method to study the daily closing prices of 2449 stocks and 29 commodities for the past 15 years. As a result, it was found that both stock prices and commodity prices have multifractal characteristics, and that the multifractal spectrum of stock prices is significantly narrower than the multifractal spectrum of commodity prices, indicating that commodity price fluctuations have a strong correlation. Drożdż [7] forecasted the two world-leading stock market indices such as the S&P500 and NASDAO based on the long-term records and results showed that multifractal features existed. However, the characteristics evolve through shapes, most of them are strongly asymmetric. Shao and Wang [8] applied the MF-DFA to study the difference in financial market of South Korea before and after South and North Korean leaders' meeting on April 27, 2018. The results showed that multifractality existed in all time intervals for both the markets, and the multifractal characteristics after the meeting were stronger, which indicated that before the meeting, the financial markets have a higher market efficiency, and decreased after the meeting. They concluded that the meeting of South and North Korean leaders since April 27, 2018 was a possible interpretation for changes of efficiency in South Korean financial markets. Moreover, Lahmiri and Bekiros [9] investigated the multifractal chaotic dynamics of Islamic and Green crypto-currency series in econophysics literature. Their findings indicated that the price, volatility and volume series of Islamic and green crypto-currencies embed high persistence compared to the conventional crypto-currencies. Chen [10] analyzed the time series of the WTI and Brent Crude oil spot prices series from 1987 to 2008. The analysis results indicated that range Hurst exponent and Renyi exponent were changed by the steps change of time series, and there were multifractal characteristics in international crude oil spot price systems. Recently, many studies on air pollutants have been conducted and the results have suggested that the increased levels of PM<sub>2.5</sub> are associated with a higher mortality and some negative effects on the lungs [11]. Wang et al. [12] investigated the implications of "Haze Special Law" on air quality in South Korea. The authors found that the multifractality after the implementation of the law was weaker than that before the law for all the cities, which shows that the enactment of the law has played a role in improving the efficiency of air pollution control in South Korea. There were also some multifractal research related to meteorological factors studied in [13, 14].

Moreover, 1D MF-DFA has been widely applied in the field of heart rate dynamics in the past decades. By analyzing the multifractal spectrum of human heart rate, many important conclusions have been used in clinical diagnosis. Ivanov et al. [15] found the multifractal properties and nonlinear characteristics of the heart rate of healthy human heartbeat and encode it in the Fourier phases. In addition to adults, MF-DFA is also used to provide elaborated and detailed analysis of FHR [16].

In addition, many achievements have been made in the application of fractal theory in image field. Especially, for image segmentation field, past works [17–20] have shown multiple ways to make this method feasible. Research shows that it is not enough to describe complex non-linear systems by only one fractal dimension. Local conditions are very important in the formation of complex structures. In order to further understand the role of local conditions in the formation of fractal bodies, a multifractal method is proposed in [21]. Based on multifractal spectrum,