Journal of Information and Computing Science
J. Info. Comput. Sci., Vol. 18, No. 2, pp. 101-116

DOI: 10.4208/JICS-2023-007
December 2023

Mathematical Formulation and Investigation of Contamination in the System of Lakes: A Comparative Study

Muhammad Hamid[1], Xiong Ying[2], Mubashar Ali Akbar[3], and Muhammad Usman[4,*]

[1] *School of Mathematics and Statistics, and Center for Applied Mathematics of Jiangsu Province, Nanjing University of Information Science and Technology, Nanjing 210044, China;*

[2] *Reading Academy, Nanjing University of Information Science and Technology, Nanjing 210044, China;*

[3] *School of Computer and Software, Nanjing University of Information Science and Technology, Nanjing 210044, China;*

[4] *School of Mathematical Sciences, Jiangsu University, Zhenjiang 212013, China*

Abstract. The contamination in water bodies is a big threat to the environment to control the water pollution, a model study is conducted. The pollution model for a system of three lakes that are interconnected by channels is taken into account. Three input models (periodic, exponentially decaying, and linear) were solved by employing a variational iteration technique coupled with various types of multipliers. Identification of exact multipliers for nth-order differential equations is also presented. All said models were examined mathematically. We noticed that exact Lagrange multipliers in the correction functional bring more accurate and efficient results that have been deliberated graphically. The results obtained via variational iteration method (VIM)-λ_E, already published work and the Runge-Kutta method of order four provides an excellent agreement. Additionally, the use of exact Lagrange multipliers with other techniques may be extended to other dynamical problems.

AMS subject classifications: 65L06, 34C20

Key words: Pollution, System of lakes model, Exact Langrage multipliers, Variational iteration method, Differential transform method, RK-4.

1 Introduction

*Corresponding author. *Email addresses:* **mmusmans@pku.edu.cn** (M. Usman).

Nowadays, a very severe issue for our environment is pollution. Pollution is the presentation of impurities into the indigenous habitat that cause antagonistic change. Pollution can appear as synthetic substances or vitality, for example, heat, light, and noise. Pollutions, the segments of contamination, can be either remote energies/substances or normally happening contaminants. The contamination in water bodies like rivers, lakes, aquifers, canals, oceans, and groundwater is known as water pollution. This type of environmental degradation happens when contaminations are indirectly or directly released into water forms without sufficient treatments to eliminate dangerous compounds. It is important to save our environment through proper planning for this threat. Monitoring of this issue using differential equations is probable now. A model [1] was introduced to study the pollution of a system of lakes. Three altered input mathematical simulations were presented to observe the pollution in a lake.

Theoretical investigation of physical and dynamical systems has huge importance in engineering and sciences. Mathematical modeling and solutions of these modeled problems via mathematical algorithms are very significant. In the past, many researchers devoted their attention to develop new algorithms or to extend existing techniques. Analytical [1-5], numerical [6-8] and wavelets [9-10] based algorithms have been utilized to explore differential equations. A lot of research work has been done using analytical techniques, which is the evidence analytical techniques [2-5] provide better approximations for physical models. An analytical technique that provides rapid convergence is presented by a Chinese mathematician He [11]. A careful survey of the literature revealed that VIM is more effective and proficient in getting the numerical and analytic solutions of nonlinear models. Ramous [12], Tatari, and Dehghan [12] proved the convergence of the variational iteration method. Later, various scholars used this method to find the solution to nonlinear problems. He constructs the approximate solution of strongly nonlinear equations [14-15]. The cited work [16-19] endorses that VIM is a comparatively better technique for obtaining the solution of nonlinear problems.

Many modifications were made in VIM to find appropriate solutions to various types of physical problems. Noor and Mohyud-Din [20] developed the elegant coupling of Adomian's polynomials with correction functional named it as modified variational iteration method (MVIM). Nadjafi and Ghorbani [21] introduced another modification in VIM. Herisanu and Marinca's modification was much more attractive, where the variational iteration method coupled with the least squares technology [22]. Yilmaz and Inc constructed a variational iteration algorithm, wherein they introduced an auxiliary parameter [23]. Hosseini et al. [24] proposed a new algorithm using an auxiliary parameter in the variational iteration method. Coupling of Taylor's series with correction functional cited in [25]. Recently, the solution of delay differential equations was tackled by VIM [26].

J. L. Lagrange was the pioneer who introduced the concept of Lagrange multiplier. He solved an isoperimetric problem in the calculus of variations. It is a constrained variational principle to maximize a closed area with a fixed perimeter. Lagrange premeditated Euler's result on the said problem and referred him to his results. Later, in