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Lie symmetries, Conservation laws and Solutions for (4+1)-dimensional time fractional KP equation with variable coefficients in fluid mechanics

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Abstract. In recent years, high-dimensional fractional equations have gained prominence as a pivotal focus of interdisciplinary research spanning mathematical physics, fluid mechanics, and related fields. In this paper, we investigate a (4+1)-dimensional time-fractional Kadomtsev-Petviashvili (KP) equation with variable coefficients. We first derive the (4+1)-dimensional time-fractional KP equation with variable coefficients in the sense of the Riemann-Liouville fractional derivative using the semi-inverse and variational methods. The symmetries and conservation laws of this equation are analyzed through Lie symmetry analysis and a new conservation theorem, respectively. Finally, both exact and numerical solutions of the fractional-order equation are obtained using the Hirota bilinear method and the pseudo-spectral method. The effectiveness and reliability of the proposed approach are demonstrated by comparing the numerical solutions of the derived models with exact solutions in cases where such solutions are known.

AMS subject classifications: 22E47, 35G20, 35B10

Key words: Time fractional equation, Conservation laws, Hirota bilinear method, Pseudo-spectral method.

1 Introduction

In recent years, the research on high-dimensional integrability has gradually become a new hot topic [1, 2]. Many high-dimensional equations can describe extremely complex physical phenomena in nature. The study of high-dimensional nonlinear equations plays

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an important role in helping us understand some facts that cannot be understood by ordinary observation. In the previous study of nonlinear partial differential equation, many scholars realized the importance of high-dimensional nonlinear partial differential equation, and spent a lot of time to find the appropriate high-dimensional nonlinear partial differential equations [3–5].

KP equation was discovered in the study of nonlinear wave theory in weakly dispersive media by Kadomtsev and Petviashvili, physicists of the former Soviet Union. It possesses a broad physical background and significant applications in plasma physics, gas dynamics, and fluid mechanics. There are few studies on variable coefficient KP equation. The variable coefficient KP equation can describe the actual surface wave better than the constant coefficient KP equation. It can deal with the concrete situation of the surface wave entering the sea or ocean through the canyon when the width, depth and density change constantly. In recent years, with the high-dimensional nonlinear problems gradually become a hot topic, some (3+1)-dimensional KP equations [6–9] and (4+1)-dimensional KP equations have appeared. Fan et al. [10] first proposed a (4+1)-dimensional variable-coefficient KP equation in 2021, deriving lump solutions and interaction solutions including rogue waves and kink waves. Later, Zhu et al. [11] made some additions to the solutions of this equation. The equation has the form

$$\begin{aligned} &f(t)u_x^2+f(t)uu_{xx}+g(t)u_{xxxx}+h_7(t)u_{ss}+h_6(t)u_{zz}+h_5(t)u_{yy} \\ &\quad +h_4(t)u_{xs}+h_3(t)u_{xz}+h_2(t)u_{xy}+h_1(t)u_{xx}+u_{xt}=0, \end{aligned} \tag{1.1}$$

where $u=u(x,y,s,z,t)$. $f(t)$ and $g(t)$ represent the nonlinearity and dispersion, respectively. $h_1(t)-h_4(t)$ stand for the perturbed effects. $h_5(t)-h_7(t)$ describe the disturbed wave velocities.

While the above equation is of integer order, fractional-order phenomena exist in natural systems as fundamental physical manifestations. At present, growing people pay attention to fractional equations, and the theory of fractional calculus is becoming more and more mature. Therefore, in this article, we try to extend the integer order (4+1) dimensional KP equation with variable coefficients to the fractional order form, and study the fractional equation. The time fractional form of the equation mentioned above has been derived for the first time using the semi-inverse method and the variational method [12]. This derivation has provided a more general significance to the equation. In our study, we focus on analyzing the symmetry, conservation laws, exact solutions, and numerical solutions of this equation.

Symmetry and conservation laws are very important for the study of partial differential equations. Recent advancements have been made in the investigation of non-classical Lie symmetries associated with partial differential equations. It is obvious that studies will be carried out on its application to fractional differential equations in the near future. Li's logarithmic method [13, 14] provides a robust framework for deriving analytical solutions to nonlinear partial differential equations. It was proposed by Markus Surface Li, a Norwegian mathematician. Gulsen [15] applied the technique that corresponds to nonclassical symmetries to obtain new solutions to evolutionary-type equations. The nature