ACADEMIC
World Academic Union

ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 6, No. 4, 2011, *pp*. 261-268

The Computation of Common Infinity-norm Lyapunov Functions for Linear Switched Systems

Zheng Chen[1,2], Yan Gao[1]

[1]Business School, University of Shanghai for Science and Technology, Shanghai 200093, China
[2]Faculty of Science, Ningbo University of Technology, Ningbo 315016, China

(*Received April 19, 2011, accepted May 17, 2011*)

Abstract. This paper studies the problem of the computation of common infinity-norm Lyapunov functions. For a set of continuous-time LTI systems or discrete-time LTI systems whose system matrices are upper triangular form or lower triangular form, it is proved that there exist common infinity-norm Lyapunov functions for them. Then four algorithms of computing common infinity-norm Lyapunov functions are presented. Finally, several examples are listed.

Keywords: Common Lyapunov functions, infinity-norm, switched systems

1. Introduction

A switched systems is one that combines continuous (or discrete) dynamics with a logic-based switching mechanism that determines abrupt mode switches in the system, operation at various points in time[1]. Most research has been devoted to the stability of switched systems[2-4]. As we know, Lyapunov functions play an important role in the stability theory of control systems for some time. In view of this, a considerable amount of recent work has focused on applying similar ideas to switched systems. Most recently many authors have derived conditions for the stability of linear switched systems based on the existence of common Lyapunov functions for their constituent systems[5-7]. For numerical and practical reasons, common quadratic Lyapunov functions are usually chosen[8-10]. However，quadratic Lyapunov functions can be too conservation and efforts have been devoted to the development of other types of common Lyapunov functions. Common infinity-norm Lyapunov functions are important one which have been studied to considerable extent[11-12] . How to compute common Lyapunov functions is of importance because this will provide some meaningful results of control systems. In this paper, we give algorithms of computing common infinity-norm Lyapunov functions.

2. Preliminaries

Throughout this note the following notation is used:

Let R^n denote real n dimensional space.

$R^{m\times n}$ denotes the set of $m\times n$ real matrices.

A^{-1} is the converse of $A\in R^{m\times n}$.

The l_p norms $\|x\|_p$, $1\le p\le\infty$, are defined by

$$\|x\|_p=(\sum_{i=1}^{n}|x_i|^p)^{1/p},$$

And

This work was supported by National Science Foundation of China (under grant: 10671126，40901241), Shanghai leading discipline project(under grant:S30501) ,Scientific research plan
projects of Zhejiang Education Department (under grant:Y201016275)

$$\|x\|_\infty = \max_{1\le i\le n} |x_i| .$$

The infinity norm of matrix $A \in R^{n\times n}$ is defined by

$$\|A\|_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}|$$

Consider a family of linear systems

$$\dot{x} = A_i x\,, \quad x \in R^n\,, \quad A_i \in R^{n\times n}\,, \quad i = 1,\cdots,N\,. \tag{1}$$

Definition 1 A function

$$V(x) = \|Wx\|_\infty , W \in R^{m\times n}$$

is said to be a common Lyapunov function of the linear systems (1) if there exist matrices

$$Q_i \in R^{m\times m},\ i = 1,\cdots,N$$

such that

$$WA_i = Q_i W \tag{2}$$

and

$$q_{jj}^i + \sum_{\substack{k=1\\k\neq j}}^{m} |q_{jk}^i| < 0 \tag{3}$$

for all $1 \le i \le N$, $1 \le j \le m$. q_{jk}^i – entries of the matrix Q_i .

Given a set of stable discrete-time LTI systems described by the following equations

$$x(t+1) = A_i x(t)\,, x \in R^n\,, A_i \in R^{n\times n}\,, i = 1,\cdots,N\,. \tag{4}$$

Definition 2 The function of the vector norm form

$$V(x) = \|Wx\|_\infty , W \in R^{m\times n}, m \ge n,\ Rank(W) = n$$

is said to be a common infinity-norm Lyapunov function for the set of systems(4) if there exist matrices $Q_i \in R^{m\times m},\ i = 1,\cdots,N$ such that w have the matrix relations

$$WA_i = Q_i W \tag{5}$$

and

$$\|Q_i\|_\infty < 1 \tag{6}$$

for all $1 \le i \le N$.

3. Computation of common infinity-norm Lyapunov functions

Let us consider A_i , $i = 1,\cdots,N$ in (1) or (4) have the form as follows

$$\begin{pmatrix} a_{11}^i & a_{12}^i & \cdots & a_{1n}^i \\ 0 & a_{22}^i & \cdots & a_{2n}^i \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn}^i \end{pmatrix}$$

or

$$\begin{pmatrix} a_{11}^i & 0 & \cdots & 0 \\ a_{21}^i & a_{22}^i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}^i & a_{n2}^i & \cdots & a_{nn}^i \end{pmatrix}.$$

The following we give the theorem below.