ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 6, No. 4, 2011, *pp*. 279-286

Synchronization Criterions between Two Identical or Different Fractional Order Chaotic Systems

Yuhua Xu [1, 2, 3, +], Wuneng Zhou [3], Jianan Fang [3], Ling Pan [3] and Wen Sun [4]

[1] Department of Mathematics and Finance, Yunyang Teachers' College, Hubei, Shiyan 442000, PR. China
[2] Computer School of Wuhan University, Wuhan, 430079, PR.China
[3] College of Information Science and Technology, Donghua University, Shanghai 201620, PR. China

[4] School of Mathematics and Information, Yangtze University, Hubei, Jingzhou 434023, PR. China

(*Received February 1, 2011, accepted March 12, 2011*)

Abstract. This paper discusses chaos synchronization between two identical or different fractional order chaotic systems. Base on linear matrix inequality, two new synchronization criterions are constructed by which it is proved that two identical (Lü system) or different (Lü and Chen systems) fractional order chaotic systems are synchronized using the simple linear feedback control laws. Finally, simulations results show the method is effective.

Keywords: Synchronization, Fractional order, Linear matrix inequality

1. Introduction

Since Pecora and Carroll established a chaos synchronization scheme for two identical chaotic systems with different initial conditions[1], variety of method and techniques have been proposed for the control and synchronization of chaotic systems such as linear and nonlinear feedback synchronization[2-6], impulsive synchronization [7-8], adaptive synchronization [9-12], observer based control method[13-15], and etc.

Fractional calculus deals with derivatives and integration of arbitrary order [16–18] and has deep and natural connections with many fields of applied mathematics, engineering and physics. Fractional calculus has wide range of applications in control theory [19], Furthermore, recently, study of chaos synchronization in fractional order dynamical systems and related phenomena is receiving growing attention , some synchronization-based strategies have been devised to synchronize fractional chaotic systems [20-25]. In Ref. [26], the synchronization of fractional-order chaotic systems has been presented. In Refs. [27-28], nonlinear control are employed to synchronize two fractional-order chaotic systems. In Ref. [29], Synchronization of N-coupled fractional-order chaotic systems with ring connection has been reported. In Refs. [30-32], Synchronization of different fractional order chaotic systems using active control has been discussed. In Ref. [33], the stability of the fractional order unified chaotic system has been studied. In this paper, base on linear matrix inequality, two new synchronization criterions are constructed by which it is proved that two identical (Lü system) or different (Lü and Chen systems) fractional order chaotic systems are synchronized.

This work is presented as follows: Section 2 describes mathematical preliminaries and model. Chaos synchronization between two identical fractional order Lü systems in Section 3. Section 4 handles chaos synchronization between Lü and Chen systems of fractional order. Section 5 gives the conclusion of the paper.

2. Mathematical preliminaries and model

In this section, we give some useful mathematical preliminaries.

The mathematical definition of fractional derivatives and integrals has been the subject of several different approaches. The definition of fractional integrals by Grunwald–Letnikov and Riemann–Liouviller

[+] Corresponding author. Tel.: 07198846038; fax: +86 7198846038.
E-mail address: yuhuaxu2004@163.com, wnzhou@163.com.

is as follows [34]:

$$I_{\mu}^{\lambda}x(t)=\frac{1}{\Gamma(\lambda)}\int_{\mu}^{t}(t-\tau)^{\lambda-1}x(\tau)d\tau\,, \tag{1}$$

where $(\mu,t)\in R^2$, $\mu<t$, $0<\lambda<1$, $\Gamma(\cdot)$ is Gamma function, $\Gamma(\lambda)=\int_0^{\infty}\vartheta^{\lambda-1}e^{-\vartheta}d\vartheta$, and $\Gamma(z+1)=z\Gamma(z)$.

The definition of fractional derivatives is as follows:

$$D_{\mu}^{\lambda}x(t)=\frac{d}{dt}[I_{\mu}^{\lambda}x(t)]=\frac{1}{\Gamma(1-\lambda)}\frac{d}{dt}\int_{\mu}^{t}(t-\tau)^{-\lambda}x(\tau)d\tau\,. \tag{2}$$

The fractional order Lü system and Chen system [35-36] are described by (see Fig.1-2)

$$\begin{cases}\dfrac{d^{\alpha_1}x_1}{dt^{\alpha_1}}=36(x_2-x_1),\\ \dfrac{d^{\alpha_2}x_2}{dt^{\alpha_2}}=-x_1x_3+20x_2,\\ \dfrac{d^{\alpha_3}x_3}{dt^{\alpha_3}}=x_1x_2-3x_3,\end{cases} \tag{3}$$

and

$$\begin{cases}\dfrac{d^{\alpha_1}y_1}{dt^{\alpha_1}}=35(y_2-y_1),\\ \dfrac{d^{\alpha_2}y_2}{dt^{\alpha_2}}=(28-35)y_1+28y_2-y_1y_3,\\ \dfrac{d^{\alpha_3}y_3}{dt^{\alpha_3}}=y_1y_2-3y_3.\end{cases} \tag{4}$$

3. Chaos synchronization between two identical fractional order Lü systems

In this section we study the synchronization between two identical fractional order Lü systems, we define the drive (master) and response (slave) systems as follows:

$$\begin{cases}\dfrac{d^{\alpha_1}x_1}{dt^{\alpha_1}}=36(x_2-x_1),\\ \dfrac{d^{\alpha_2}x_2}{dt^{\alpha_2}}=-x_1x_3+20x_2,\\ \dfrac{d^{\alpha_3}x_3}{dt^{\alpha_3}}=x_1x_2-3x_3,\end{cases} \tag{5}$$

and

$$\begin{cases}\dfrac{d^{\alpha_1}y_1}{dt^{\alpha_1}}=36(y_2-y_1)+u_1,\\ \dfrac{d^{\alpha_2}y_2}{dt^{\alpha_2}}=-y_1y_3+20y_2+u_2,\\ \dfrac{d^{\alpha_3}y_3}{dt^{\alpha_3}}=y_1y_2-3y_3+u_3.\end{cases} \tag{6}$$

We define control functions u_i as

$$u_1=-k_1e_1,\,u_2=-k_2e_2,\,u_3=-k_3e_3\,, \tag{7}$$

where $k_1>0$, $k_2>0$, $k_3>0$.

The error functions as