ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 6, No. 4, 2011, *pp*. 303-313

Fast Iterative Method-FIM: Application to the Convection-Diffusion Equations

H.SaberiNajafi, S.A.Edalatpanah[+]

Departemant of Mathematics, Islamic Azad University, P. Code 1616, Lahijan, Iran
Department of Mathematics, Faculty of Sciences, Guilan University, P.O. Box 41335-1914, Rasht, Iran

(Received July 9, 2011, accepted September 12, 2011)

Abstract. In this paper we present a new algorithm for solving linear systems by iterative method. In this method the rate of convergence has been improved well. The results show that this new algorithm converges faster than AOR, SOR, SSOR, etc .and works with high accuracy. The Main difference between these methods with the others is that uses affine combinations and also less parameter. Besides, we have shown that a divergent process can be converges by using FIM method. Finally the method is tested by some numerical experiments.

Keywords: iterative method, spectral radius, AOR, SOR, SSOR, Convection-Diffusion equation

1. Introduction

The iterative methods for solving a linear system are well known and some of them like MAOR, AOR, SSOR and SOR, are very popular [1-4].In this article a new iterative method presents which uses less parameter and converges faster than the other methods. There are examples of divergent iterative methods, which converge when using this new algorithm. Let

$$Ax=b \tag{1.1}$$

Where $A \in R^{n \times n}$ & $b, x \in R^n$ and A is nonsingular. The basic iterative formula for (1.1) is

$$x^{(i+1)} = M^{-1}Nx^{(i)} + M^{-1}b \qquad i = 0,1,\dots \tag{1.2}$$

Where x^0 is an initial guess .if A is split in to A=M-N, where M is nonsingular, then the basic iterative method for solving (1.1) is (1.2) .this iterative process converges to the unique solution $X = A^{-1}b$ for any initial vector value $x^0 \in R^n$ if and only if $\rho(\underbrace{M^{-1}N}_{B}) < 1$ where B=M^{-1} N is called iteration matrix .Suppose diag(A)=I and A=I-L-U, Where L and U are strictly lower and strictly upper triangular part of A, respectively. Now for Jacobi iterative method M=I, N=L+U and for Gauss-Seidel M=I-L, N=U.

That is

$$\begin{aligned} &jacobi: && x^{(i+1)} = b + Lx^{(i)} + Ux^{(i)} \\ &gauss-seidel && x^{(i+1)} = b + Lx^{(i+1)} + Ux^{(i)} \end{aligned} \tag{1.3}$$

In the new method, which we call FIM, not only the new iteration values are used, the previous values also used for computing the next values. In other words we have

$$\begin{cases} Ax = b \\ x^{(i+1)} = b + L\{(1-k)x^{(i+1)} + kx^{(i)}\} + Ux^{(i)} \qquad ;k \in R \end{cases} \tag{1.4}$$

Difinition1.1.[5] if $s = \{x^0, x^1, \dots, x^n\}$, then the set of all affine combinations of s is defined as

[+] Corresponding author. E-mail addresses: hnajafi@guilan.ac.ir H. Saberi Najafi), saedalatpanah@gmail.com (S.A.Edalatpanah).

$$A(s)=\{x \mid x=\sum_{j=0}^{n}\lambda_j x^j \quad ;\sum_{j=0}^{n}\lambda_j =1 \quad ;\lambda_j \in R\} \tag{1.5}$$

2. Basic idea of FIM

In the Gauss – Seidel process, i.e. $x^{(i+1)} = b + Lx^{(i+1)} + Ux^{(i)}$ We would like to improve $Ux^{(i)}$ and substitute the better value instead. There are two suggestions for implementing the above goal:

i.statistical method

Using variance and standard deviation $Ux^{(i)}$ can be changed to $U(x^{(i)}+\delta(x))$ where, $\delta(x)$ is standard deviation. (Confidence interval, statistical tests)

ii.prediction method

2.1. Two steps predictions (the prediction method with two sub iteration)

This process like SSOR contains two half iterations in each step, but in our method first the better value of $Ux^{(i)}$ provides and then improved more, i.e.

$$\begin{aligned} x^{(i+\frac{1}{2})} &= b + Lx^{(i+\frac{1}{2})} + Ux^{(i)} \\ x^{(i+1)} &= b + Lx^{(i+1)} + Ux^{(i+\frac{1}{2})} \end{aligned} \tag{2.1}$$

2.2. Four steps predictions

In this process each iterations involves quarter iterations i. e,

$$\begin{cases} x^{(i+\frac{1}{4})} = b + Lx^{(i+\frac{1}{4})} + Ux^{(i)} \\ x^{(i+\frac{2}{4})} = b + Lx^{(i+\frac{2}{4})} + Ux^{(i+\frac{1}{4})} \\ x^{(i+\frac{3}{4})} = b + Lx^{(i+\frac{3}{4})} + Ux^{(i+\frac{2}{4})} \\ x^{(i+1)} = b + Lx^{(i+1)} + Ux^{(i+\frac{3}{4})} \end{cases} \tag{2.2}$$

Example 2.1 Let

$$A=\begin{bmatrix} 1 & -.7 & -.3 & 0 & -.2 & -.2 \\ 0 & 1 & 0 & -.4 & -.1 & 0 \\ -.1 & 0 & 1 & -.6 & -.1 & -.4 \\ -.3 & -.3 & -.2 & 1 & -.1 & -.2 \\ 0 & -.2 & 0 & -.1 & 1 & 0 \\ -.9 & 0 & 0 & 0 & -.2 & 1 \end{bmatrix}$$

The standard iterative methods and the FIM method are used for this matrix. The spectral radius in each case computed and compared with each other in Table 1. As the results show the spectral radius of FIM is smaller than the others which means faster convergence.

Table1 shows the results of example2.1

Iterative method	Spectral radius
Jacobi	0.9859
Gauss-Seidel	0.9760
Symmetric Gauss- Seidel	0.9632
Two steps FIM	0.9526
Four steps FIM	0.9127

To show a proof for the FIM method and to describe why it works well, we consider the following: