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Conceptual Category of Knowledage and Its Correlation-based Metrizable Space

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Abstract. The phenomenon was understood as the basic element to constitute knowledge in this study, where the granular structure was taken as the basic structure of phenomenon formalized representation. Based on the isomorphism between formal concept analysis and rough set theory, the granular structure model was established in conceptual category of phenomenon, and the correlation among concept categories was given as well as correlation distance, proving that the concept element space induced by correlation distance Θ was a metrizable space. Based on the conclusions in this study, the knowledge representation model can be established more completely than conceptual model in semantic description, providing a basis for phenomenon formalization research by using the topologic method.

Keywords: Knowledge Representation; Phenomenon; Formal Concept Analysis; Rough Set; Metric Space

1. Introduction

In knowledge engineering, semantic Web, knowledge discovery, information retrieval and other fields, it is required to conduct representation and description on knowledge as well as establish the formalized conceptual model. However, the description on knowledge is derived from that on “phenomenon” in natural, social and artificial systems. The human cognitive process on objective world was generalized as granulation, organization, and causal reasoning by Zadeh [1]. This process can also be mapped as the phenomenon structure in objective world. The phenomenon was taken as a research object in this study, and its structure contains the granular structure of categories, their correlation and relevance logic. Among them, the categories of phenomenon include the conceptual category of its semantics and the time - state category as the description on its movement and evolution basis. The correlation is able to link loose individual phenomena, providing a basis for the formation of knowledge with a core and boundaries. While, specific constraints are given to the correlation among phenomena by relevance logic. This study was carried out on granular structure of conceptual category in phenomenon as well as its correlation.

According to good definition on “concept” and its representative form consistent with granular structure in knowledge discovery and other fields proposed by formal concept analysis (FCA) theory [2,3] as well as the process on incomplete and insufficient knowledge in rough set (RS) theory [4,5,9], the descriptions on conceptual category of phenomenon were carried out by combining both theories and then incorporated into granular structure. Meanwhile, the correlation and correlation distance were also studied. Based on correlation distance, the definition of concept element space was given and proven as a metrizable space.

2. Phenomenon and the formalized representation of its categories

2.1. Granular structure

Human brain is a hierarchical structure, where the human knowledge structure is determined as a hierarchy by the formation and learning process of knowledge. Then, the phenomenon in objective world is taken as the mapping of human knowledge structure also with the same hierarchy. Such structure is considered as “granular structure” in view of granular computing theory [6,7], where the three basic attributes are satisfied[8], including (1) context attribute indicating the existence of a grain in special environment; (2) internal attribute reflecting elemental interaction in a grain; (3) external attribute reflecting the interaction between a grain and others. Based on the above attributes, the basic form of granular structure is given as below.

Definition 2.1 (granular structure): A granular structure is described with a triple $\left(E_G, I_G, R\right)$, where E_G and I_G are the sets called as granular extension and granular connotation respectively. Binary relation $R \subseteq E_G \times I_G$ is called as context relation.

In general, granular structure can be established in a special domain based on specific context. A certain constraint is defined in this domain, where granular connotation I_G is expressed by constraint rules, reflecting the general characteristics of all elements in the domain; while granular extension E_G is the element set in this domain satisfying the constraint, i.e., the set of elements covered in granular structure. However, context relation R represents the external attribute of interaction among granular structures. The formalized representation on conceptual category of phenomenon is carried out through granular structure as below.

2.2. Conceptual category

First, considering that the conceptual categories of most phenomena are ambiguous with indistinct boundaries, the knowledge domain is required with classification capacity for a rough identification and organization on scattered grains. Thereby, the indiscernibility relation of RS theory was used to process incomplete and inadequate information, and then the knowledge domain can be divided into different equivalence classes, i.e., the phenomena are classified according to different attributes or characteristics. Meanwhile, the granular structure forms of conceptual category in phenomena are given by combining the scale theory of FCA. The isomorphism between both theories is first given by the following propositions.

Proposition 2.1. Any knowledge base $\left(U, A\right)$ is given, where U is the domain and $A := \left\{B_m \mid m \in M\right\}$. $\mathrm{S}\left(U, A\right) := \left(\left(U, A, W, I\right), \left(S_B \mid B \in A\right)\right)$ is known, and S_B is the rated scale with its derived context $\left(U, N, J\right)$. γ is set to represent the mapping of object concept in the context $\left(U, N, J\right)$, and thereby, if $\left(u, v\right) \in IND\left(P\right)$ ($\left(u, v\right) \in U^2$, $P \subseteq A$), then $\gamma\left(u\right) = \gamma\left(v\right)$.

Proof. $\left[v\right]_P = \left\{v \in U \mid u\, J\left(m, n\right) \Leftrightarrow v\, J\left(m, n\right), P \subseteq A\right\}$ is set as well as $v \in \left[u\right]_P$, $\left[u\right]_P = \left[v\right]_P$ ($u, v \in U$) has been proven previously.

In fact, if $\left(u, v\right) \in IND\left(P\right)$, then $I\left(u, m\right) = I\left(v, m\right) = w$ for any $m \in P$, where $w \in W_m$ and $wI_m n$ ($n \in M_m$). According to $u\, J\left(m, n\right) \Leftrightarrow v\, J\left(m, n\right)$, $v \in \left[v\right]_P$ is known, i.e., $[u]_P \subseteq [v]_P$. In addition, if $v \in \left[v\right]_P$, $w_1, w_2 \in m\left(U\right)$ must exist, known by $u\, J\left(m, n\right) \Leftrightarrow v\, J\left(m, n\right)$, as well as $I\left(u, m\right) = w_1$, $w_1 I_m n \Leftrightarrow I\left(v, m\right) = w_2$ and $w_2 I_m n$ .Thereby, $w_1 = n \Leftrightarrow w_2 = n$, $w_1 = w_2 = n$, and $I\left(u, m\right) = I\left(v, m\right)$. According to indiscernibility relation of rough set theory, $\mathrm{c}\left(u, v\right) \in IND\left(P\right)$ exists as well as $v \in \left[u\right]_P$, i.e., $[v]_P \subseteq [u]_P$, then $\left[u\right]_P = \left[v\right]_P$. Thus, $\left[u\right]_B = \left[v\right]_B$ exists for $\forall B \in P$.

Thereby, J can be redefined, and $\left[u\right]_B = \left[v\right]_B \Leftrightarrow B\left(u\right) J \left[v\right]_B \Leftrightarrow uJ\left(B, \left[v\right]_B\right)$ exists, i.e., the derived context $\left(U, N, J\right)$ of $\mathrm{S}\left(U, A\right)$ can be expressed as $\left(U, \left\{\left(B, \left[u\right]_B\right) \mid B \in A, \left[u\right]_B \in U / B\right\}, J\right)$. $\left(u, v\right) \in IND\left(B\right) \Leftrightarrow uJ\left(B, \left[v\right]_B\right) \Leftrightarrow \gamma\left(u\right) = \gamma\left(v\right)$ can be obtained simultaneously. □

It is known by Proposition 2.1 that if a knowledge base $\left(U, A\right)$ is given, then many-valued context can be generated under the effect of scale operator, and the derived context of many-valued context is $\left(U, \left\{\left(B, \left[u\right]_B\right) \mid B \in A, \left[u\right]_B \in U / B\right\}, J\right)$, i.e., the knowledge base $\left(U, A\right)$ can be expressed with the derived context of many-valued context of formal concept. Thereby, the consistence between basic concepts of both theories is characterized in the form as a basis for establishing granular structure. Based on this