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Some Progress on Single Species Models with Nonlocal Dispersal Strategies in Heterogeneous Environments

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Abstract. In this paper, we consider a single species model with nonlocal dispersal strategy and discuss how the dispersal rate and the distribution of resources affect the total population and survival chances by summarizing some previous results and demonstrate some relevant progress. The first topic is about the monotonicity of total population upon dispersal rate. For the nonlocal model, we prove a new result, which reveals essential difference between local and nonlocal models for certain distribution of resources. Secondly, we discuss optimal spatial arrangement for survival chances and total populations. The results for both local and nonlocal models demonstrate that the concentration of resources is beneficial for species.

AMS subject classifications: 35K57, 92D25, 45K05

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1 Introduction

Understanding the combined effects of dispersal strategies and spatial heterogeneity on population dynamics is an important issue in spatial ecology [9]. For this purpose, we consider the following single species model with nonlocal dispersal strategy

$$u_t(x,t) = d\mathcal{L}u(x,t) + u(x,t)[m(x) - u(x,t)] \qquad x \in \Omega,\ t > 0, \tag{1.1}$$

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where $u(x,t)$ represents the population density of a species, $d>0$ is the dispersal rate of the species, the habitat Ω is a bounded domain in $\mathbb{R}^n$, and the function $m(x)$ is the intrinsic growth rate or carrying capacity, which reflects the environmental influence on the species u. The nonlocal diffusion operator is defined as

$$\mathcal{L}u:=\int_\Omega k(x,y)u(y)dy-a(x)u(x), \tag{1.2}$$

where the dispersal kernel function $k(x,y)\geq 0$ represents the probability to jump from one location to another one, and

- either $a(x)=1$, which corresponds to nonlocal homogeneous Dirichlet boundary condition,
- or $a(x)=\int_\Omega k(y,x)dy$, which corresponds to nonlocal homogeneous Neumann boundary condition.

For suitably rescaled kernel functions, the convergence relations between problems with nonlocal diffusion operators and those with local diffusion operators and corresponding homogeneous Dirichlet or Neumann boundary conditions are verified under certain conditions. We refer the books [2, 12], the survey paper [13] and the references therein for more details. Different from random diffusion, the nonlocal diffusion operator describes a long range process and appears commonly in different types of models in ecology. See [1,8,10,11,14,16,17,23,27–29,32,33] and the references therein.

For clarity, assume that the dispersal kernel function $k(x,y)$ satisfies

(K) $k(x,y)\in C(\mathbb{R}^n\times\mathbb{R}^n)$ is nonnegative, $k(x,x)>0$ in $\mathbb{R}^n$, and $k(x,y)$ is symmetric, i.e., $k(x,y)=k(y,x)$. Moreover, $\int_{\mathbb{R}^n}k(x,y)dy=1$.

We begin the discussion with a single logistic equation with random diffusion as follows

$$\begin{cases} u_t=d\Delta u+u[m(x)-u] & x\in\Omega,\ t>0,\\ \frac{\partial u}{\partial \nu}=0 & x\in\partial\Omega,\ t>0, \end{cases} \tag{1.3}$$

where ν denotes the unit outward normal vector on $\partial\Omega$. It is known that if $m(x)\geq 0$ is nonconstant, then for every $d>0$, the problem (1.3) admits a unique positive steady state, denoted by $\theta_{d,m}(x)$ (see, e.g., [9]). In addition, a remarkable property concerning $\theta_{d,m}(x)$ was first observed in [20]:

$$\int_\Omega \theta_{d,m}(x)dx>\int_\Omega m(x)dx \qquad \text{for all } d>0. \tag{1.4}$$

This indicates that when coupled with diffusion, different from homogeneous environment, a heterogeneous environment can support a total population larger than the total