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Analytic Insights into an Adapted Algorithm for the Score-Based Secretary Problem

Giangvuthanh Nguyen[1], Xiang Xu[1,*] and Yanxiang Zhao[2]

[1] *Department of Mathematics and Statistics, Old Dominion University, Norfolk VA 23529, USA;*
[2] *Department of Mathematics, George Washington University, Washington D.C. 20052, USA.*

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Abstract. In this paper, we study some basic analytic properties of a sequence of functions $\{S_n^{\mu,\sigma}\}$ that is directly derived in an adaptive algorithm originating from the classical score-based secretary problem. More specifically, we show that: 1. the uniqueness of maximum points of the function sequence $\{S_n^{\mu,\sigma}\}$; 2. the maximum point sequence of $\{S_n^{\mu,\sigma}\}$ monotone increases to infinity as n tends to infinity. All of the proofs are elementary but nontrivial.

AMS subject classifications: 26A06, 49K05

Key words: Secretary problem, adaptive algorithm, expected score, uniqueness of maximum points.

1 Introduction

In the classical score-based secretary problem where a decision maker is tasked with interviewing a series of candidates for a position, the goal for the decision maker is to identify and select the most qualified candidate among all the applicants. The selection process is in a sequential manner, where each candidate is interviewed one after the other. During these interviews, the decision maker assesses each candidate and assigns them a numerical score. This score represents the candidate's "value" or suitability for the position, based on factors such as qualifications, experience, and overall impression.

Upon completing an interview, the decision maker faces a critical decision for each candidate: to either accept or reject them. This decision is pivotal because of two key constraints: (1) Irreversibility of rejection: once a candidate is rejected, the decision is

*Corresponding author. *Email addresses:* x2xu@odu.edu (Xu X), gnguy002@odu.edu (Nguyen G), yxzhao@email.gwu.edu (Zhao Y)

final. The candidate cannot be recalled or reconsidered at a later stage, regardless of the quality of subsequent candidates. This feature adds a significant level of risk and complexity to the decision-making process. (2) Termination upon acceptance: conversely, if the decision maker chooses to accept a candidate, the interview process is immediately terminated. The selected candidate is deemed the best choice, and no further candidates are considered. These rules create a challenging dilemma for the decision maker. They must strategically balance the risk of rejecting potentially suitable candidates early in the process against the possibility of encountering even better candidates later on. The decision maker must thus employ a judicious combination of evaluation, forecasting, and risk assessment skills to optimize the chances of selecting the best candidate out of the entire pool.

The classical score-based secretary problem is a well-explored area in decision theory with numerous significant results. In [1], the authors discuss many cases and variations. Notably, they highlight that as the number of candidates n approach infinity, the optimal strategy is to skip the first $\frac{n}{e}$ candidates. This approach yields a probability of $\frac{1}{e}$ for selecting the top candidate, focusing solely on the candidates' ranks to maximize the probability of hiring the best one. In [2], the authors provide a comprehensive overview of the secretary problem's origins, tracing its conceptual evolution. Meanwhile, Freij and Wastlund [3] make a significant advancement by demonstrating the existence of a universal algorithm applicable to any poset, guaranteeing success with a probability of at least $\frac{1}{e}$. Preater introduces in [4] an intriguing generalization of the problem, proposing an algorithm effective across all poset sets of a given size with a positive success probability. This expansion of the problem scope adds depth to its applicability in decision-making scenarios. In an interesting twist, Bearden [5] examines the scenario where the candidate data set follows a uniform distribution. In such cases, if the decision maker prioritizes the expected value of the selected candidate, an optimal policy emerges. This strategy involves skipping the first $\sqrt{n}-1$ candidates, then selecting the next candidate who ranks highest. Most recently, Sarkar [13] considers a variant of the secretary problem, in which the employer also learns the scores of the already interviewed candidates, when making the decision after the n-th interview is over.

The field has seen other notable contributions as well. Kozik [9] introduces a dynamic threshold strategy, establishing its success probability at a minimum of n/4. Kleinberg [8] examines a unique variation where the algorithm permits selecting multiple candidates, aiming to maximize expected profit. Additionally, Korula and Pál [10] explore the problem in the context of selecting elements from specially designed graphs or hypergraphs, expanding the problem's application to more complex structures.

As a well-known best-choice problem in decision theory, the secretary problem has seen a wide range of applications to real-world situations. These applications extend to various domains, such as the house-selling problem [6, 11], dynamic and stochastic knapsack problems [7], online auction strategies [8], and online matching problems, particularly in the context of internet advertising reservation systems [10].

A recent study by Zhou et al. (2021) [12] explores a novel variation of the secretary