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Global Large Solution for the Tropical Climate Model Without Thermal Diffusion

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Abstract. In this paper, we consider the Cauchy problem of d-dimensional $(d=2,3)$ tropical climate model without thermal diffusion and construct global smooth solutions by choosing a class of special initial data (u_0,v_0,θ_0) whose L^2 norm can be arbitrarily large, and improve the previous results in [1,2].

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1 Introduction

This paper focuses on the following d-dimensional $(d=2,3)$ tropical climate model (TCM)

$$\begin{cases} \partial_t u+u\cdot\nabla u+\kappa u+\nabla p+\operatorname{div}(v\otimes v)=0, \\ \partial_t v+u\cdot\nabla v-\eta\Delta v+v\cdot\nabla u+\nabla\theta=0, \\ \partial_t\theta+u\cdot\nabla\theta+\operatorname{div} v=0, \\ \operatorname{div} u=0, \\ (u,v,\theta)|_{t=0}=(u_0,v_0,\theta_0). \end{cases} \tag{1.1}$$

Here κ and η are positive parameters, $u=u(t,x)$ and $v=v(t,x)$ stand for the barotropic mode and the first baroclinic mode of the vector velocity, respectively. $p=p(t,x)$ and

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$\theta=\theta(t,x)$ denote the scalar pressure and scalar temperature, respectively. In the case when $\kappa=\eta=0$, (1.1) was first derived by Frierson–Majda–Pauluis [3] by performing a Galerkin truncation to the hydrostatic Boussinesq equations, of which the first baroclinic mode had been originally used in some studies of tropical atmosphere, see [4–6] for more relevant background on the tropical climate model. There have been huge amount of literature on the study of (1.1) concerning the well-posedness of TCM with fractional dissipation terms, we refer to [7–15] and the references therein.

1.1 Motivation and challenges

Now we would like to point out the facts that motivate the current work. We should mention that in the case when $\theta=0$, TCM reduces to the classical MHD equations. Also, by adding the Hall term $\nabla\times((\nabla\times v)\times v)$ to the second equation of (1.1), the resulting system is called the Hall-MHD system. Recently, the MHD and Hall-MHD equations have been studied extensively mathematically concerning the local well-posedness, global small solutions, global regularity, blow-up criterions and so on. We refer to [16–23] and the references therein. Due to the lack of the incompressibility condition div $v=0$, it is much harder to establish the global well-posedness for TCM (1.1) than that for that of MHD equations. From the mathematical point of view, adding some fractional dissipation terms does bring the regularity for the system (1.1) and also significantly changes the model properties and physics. Li–Titi [24] proved the global well-posedness of 2D TCM (1.1) with two Laplacian terms Δu and Δv. Wan [25] proved the global strong solution to TCM (1.1) with additional damping term v under suitable smallness assumptions on the initial data. Later, Ma–Wan [26] removed the damping term v and obtained the global strong solution to TCM (1.1) with different smallness assumptions on the initial data. However, it seems difficult to establish the global well-posedness of TCM (1.1) with only a damping term u and zero diffusion for large initial data. Examples of smooth large initial data giving rise to global unique solutions to the MHD and Hall-MHD equations have been constructed in [27–30]. Li–Yu [1] constructed global smooth solutions to 2D TCM (1.1) with a class of special initial data, where the initial data (u_0,v_0) in $L^\infty(\mathbb{R}^2)$ can be arbitrarily large while the initial data θ_0 is small in $H^3(\mathbb{R}^2)$. Subsequently, Chen–Yuan–Zhang [2] established a class of global solutions of the d-dimensional $(d=2,3)$ TCM with additional damping term in the θ-equation by choosing a class of special initial data (u_0,v_0,θ_0) whose $H^s(\mathbb{R}^d)(s>1+d/2)$ norm can be arbitrarily large.

In this paper, we are concerned with the following natural question: Whether or not there exists a global solution to the d-dimensional $(d=2,3)$ TCM with zero thermal diffusion and some classes of large initial value (u_0,v_0,θ_0) whose $L^2(\mathbb{R}^d)$ norm can be arbitrarily large. Motivated by [1, 2, 28–30], we shall solve this problem and improve the previous results [1, 2].

Let us make some comments on the idea and the challenges we encounter. Our main idea is splitting the linearized equations from the tropical climate model (1.1). We expect