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Averaging of a Three-Dimensional Brinkman-Forchheimer Equation with Singularly Oscillating Forces

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Abstract. We consider the uniform attractors of a 3D non-autonomous Brinkman–Forchheimer equation with a singularly oscillating force

$$\frac{\partial u}{\partial t}-\gamma\Delta u+au+b|u|u+c|u|^2u+\nabla p=f_0(x,t)+\varepsilon^{-\rho}f_1\left(x,\frac{t}{\varepsilon}\right),$$

for $\rho\in[0,1)$ and $\varepsilon>0$, and the averaged equation (corresponding to the limiting case $\varepsilon=0$)

$$\frac{\partial u}{\partial t}-\gamma\Delta u+au+b|u|u+c|u|^2u+\nabla p=f_0(x,t).$$

Given a certain translational compactness assumption for the external forces, we obtain the uniform boundedness of the uniform attractor $\mathcal{A}^\varepsilon$ of the first system in $(H_0^1(\Omega))^3$, and prove that when ε tends to 0, the uniform attractor of the first system $\mathcal{A}^\varepsilon$ converges to the attractor $\mathcal{A}^0$ of the second system.

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Key Words: Brinkman-Forchheimer equation; uniform attractor; singularly oscillating external force; uniform boundedness.

1 Introduction

In this paper, we focus on the uniform attractors of a 3D non-autonomous Brinkman–Forchheimer equation describing the fluid flow in a saturated porous medium:

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$$\frac{\partial u}{\partial t}-\gamma\Delta u+au+b|u|u+c|u|^2u+\nabla p=f_0(x,t)+\varepsilon^{-\rho}f_1\left(x,\frac{t}{\varepsilon}\right),\qquad \text{in } \Omega\times(0,T), \tag{1.1}$$

$$\nabla\cdot u=0,\qquad \text{in } \Omega\times(0,T), \tag{1.2}$$

$$u|_{\partial\Omega}=0,\qquad \text{on } \partial\Omega\times(0,T), \tag{1.3}$$

$$u(x,0)=u_0(x),\qquad \text{in } \Omega, \tag{1.4}$$

where $\rho\in[0,1)$ is a fixed parameter, $\Omega\subset\mathbb{R}^3$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$, $u=u(x,t)=(u_1(x,t),u_2(x,t),u_3(x,t))$ is the fluid velocity vector, γ is the Brinkman coefficient, $a>0$ is the Darcy coefficient, $b>0$ and $c>0$ are the Forchheimer coefficients, and p denotes pressure.

At the same time, we consider the following averaged Brinkman-Forchheimer equation,

$$\frac{\partial u}{\partial t}-\gamma\Delta u+au+b|u|u+c|u|^2u+\nabla p=f_0(x,t),\qquad \text{in } \Omega\times(0,T), \tag{1.5}$$

$$\nabla\cdot u=0,\qquad \text{in } \Omega\times(0,T), \tag{1.6}$$

$$u|_{\partial\Omega}=0,\qquad \text{on } \partial\Omega\times(0,T), \tag{1.7}$$

$$u(x,0)=u_0(x),\qquad \text{in } \Omega, \tag{1.8}$$

which corresponds to the case of $\varepsilon=0$.

The external forces of the problem (1.1)-(1.4) and (1.5)-(1.8) for $0<\varepsilon<1$ and $\varepsilon=0$ are as follows:

$$f^{\varepsilon}(x,t)=\begin{cases} f_0(x,t)+\varepsilon^{-\rho}f_1\left(x,\frac{t}{\varepsilon}\right), & \varepsilon>0,\\ f_0(x,t), & \varepsilon=0. \end{cases} \tag{1.9}$$

The fluid in a saturated porous medium is very important in the petroleum industry and in the designing of air-conditioners. Thus, many researchers have studied the related equations in recent years ([1–12]), and have made a series of important advancements. However, all the previous studies on the problem (1.1)-(1.4) (when $f=f_0(x,t)$) have basically focused on the issue of continuous dependence of the solutions on the coefficients γ, b, and c (see, for instance, [1–3]). In [4], Uğurlu proved the existence of a global attractor for (1.1)-(1.4) (when $f=f(x)$, independent of t) in the phase space $(H_0^1(\Omega))^3$. In [5], Ouyang and Yang investigated the system (1.1)-(1.4) when $c|u|^2u$ is replaced by $c|u|^{\beta}u$ and $f=f(x)$. They revealed the existence of a global attractor in $(H_0^1(\Omega))^3$ when $1<\beta\le\frac{4}{3}$. In [6], Wang and Lin showed that the system (1.1)-(1.4) has a global attractor in $(H^2(\Omega))^3$ when the external forcing term $f=f(x)\in(L^2(\Omega))^3$. In [7], Song and Qiao proved the existence and structure of a uniform attractor in $(H_0^1(\Omega))^3$ for the processes associated with a fluid when the external force $f_0(x,t)$ is translation-compact in $L^2_{\rm loc}(\mathbb{R},(L^2(\Omega))^3)$. In [8], using the artificial compressibility method, Zhao and You studied the approximation