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Global Well-Posedness and Incompressible Limit of the Hall-Magnetohydrodynamic System in a Bounded Domain

MU Yanmin, WANG Wenkang and ZHANG Jing*

School of Applied Mathematics, Nanjing University of Finance and Economics, Nanjing 210023, China.

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Abstract. In this paper we firstly prove the global well-posedness for the compressible Hall-magnetohydrodynamic system in a bounded domain when the initial data is small. On this basis, we continue to study the convergence of the corresponding equations with the well-prepared initial data as the Mach number tends to zero.

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1 Introduction

In this paper we consider the compressible Hall-magnetohydrodynamic (Hall-MHD) system (see [1]) in a bounded domain Ω as:

$$\partial_t\rho+\operatorname{div}(\rho u)=0, \tag{1.1}$$

$$\partial_t(\rho u)+\operatorname{div}(\rho u\otimes u)+\nabla \mathrm{P}(\rho)-\mu\Delta u-(\lambda+\mu)\nabla\operatorname{div}u=\operatorname{curl}H\times H, \tag{1.2}$$

$$\partial_t H+\nabla\times\Big(H\times u+\frac{\operatorname{curl}H\times H}{\rho}\Big)=-\nabla\times\operatorname{curl}H,\qquad \operatorname{div}H=0. \tag{1.3}$$

Here the unknowns are ρ, $u=(u_1,u_2,u_3)\in\mathbb{R}^3$, $H=(H_1,H_2,H_3)\in\mathbb{R}^3$ denoting the density, the velocity field and the magnetic field, respectively. Let $\mathrm{P}(\rho)$ be a C^1 smooth function

*Corresponding author. *Email addresses:* `yminmu@126.com` (Y. M. Mu), `718642128@qq.com` (W. K. Wang), `jingzhang0209@126.com` (J. Zhang)

of ρ, we further assume that $P'(\cdot)>0$ and $P(0)=0$, $P'(1)=1$ for simplicity. The spatial domain Ω is a bounded domain with smooth boundary $\partial\Omega$ in $\mathbb{R}^3$. The parameters μ, λ denote the shear and bulk viscous coefficient respectively. For simplicity, let λ and μ be positive constants.

For the system (1.1)-(1.3), the initial and boundary conditions are prescribed as:

$$u\cdot n=0,\quad \mathrm{curl}u\times n=0,\quad H\cdot n=0,\quad \mathrm{curl}H\times n=0,\qquad (x,t)\in\partial\Omega\times(0,T), \tag{1.4}$$

$$(\rho,u,H)(x,0)=(\rho_0,u_0,H_0)(x),\qquad x\in\Omega. \tag{1.5}$$

In many physical phenomena such as magnetic reconnection in space plasmas, star formation, neutron stars and geo-dynamo, the Hall-MHD equtions are involved [2]. If we neglect the term $\frac{(\nabla\times H)\times H}{\rho}$ in (1.3) which reflects the Hall effect, then the system (1.1)-(1.3) is known as the compressible MHD system. There have been many research results on MHD, which can be found in [3–7] and the references cited therein. Li and Wang [4] studied the local strong solution, Hu-Wang [5] and Suen-Hoff [6] obtained the global weak solutions for large initial data and small initial data respectively. The low Mach number limit to the compressible isentropic MHD equations was studied in [7] in the whole space $\mathbb{R}^3$ or the torus $\mathbb{T}^3$. For the bounded domain $\Omega\subset\mathbb{R}^2$, Dou-Jiang-Ju [3] and Dou-Ju [8] proved the incompressible limit. Fan, Li and Nakamura [9] generalized the results to three-dimensional case.

Next, let's review existing results on the Hall-MHD system. For incompressible case, the results of well-posedness can be referred to Acheritogaray-Degond-Frourelle-Liu [1], Dumas-Sueur [10] on global weak solutions, Chae-Degond-Liu [11] on the local smooth solutions. For compressible case, Fan-Alsaedi-Hayat-Nakamura-Zhou [12] studied the global existence and time decay rate of smooth solutions. The first author [13] established the zero Mach limit of the system (1.1)–(1.3) on $\mathbb{R}^3$.

In this present paper, we mainly establish the global existence of strong solutions and then show the convergence of the system (1.1)–(1.5) as Mach number tends to zero, namely we generalize some results in [3,8,9] to the Hall-MHD system.

Firstly, there are some scaling transformations on the functions(ρ,u,H) as the following:

$$\rho(x,t)=\rho^\epsilon(x,\epsilon t),\quad u(x,t)=\epsilon u^\epsilon(x,\epsilon t),\quad H(x,t)=\epsilon H^\epsilon(x,\epsilon t),$$

where ϵ is the (scaled) Mach number, We usually consider the density variations is small enough, i.e.,

$$\rho^\epsilon:=1+\epsilon q^\epsilon.$$

So we can rewrite the system (1.1)-(1.5) as:

$$\partial_t q^\epsilon+u^\epsilon\cdot\nabla q^\epsilon+\frac{(1+\epsilon q^\epsilon)}{\epsilon}\mathrm{div}u^\epsilon=0, \tag{1.6}$$

$$\rho^\epsilon\partial_t u^\epsilon+\rho^\epsilon u^\epsilon\cdot\nabla u^\epsilon+\frac{1}{\epsilon}P'(1+\epsilon q^\epsilon)\nabla q^\epsilon+\mu\mathrm{curl}^2u^\epsilon-(\lambda+2\mu)\nabla\mathrm{div}u^\epsilon=\mathrm{curl}H^\epsilon\times H^\epsilon, \tag{1.7}$$