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Dynamics for the Stochastic Plate Equations of Kirchhoff Type on $\mathbb{R}^n$

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Abstract. In this paper, we study mainly the long-time behavior of the stochastic plate equations of kirchhoff type with nonlinear damping on unbounded domains. Due to the lack of compactness of Sobolev embeddings on unbounded domains, pullback asymptotic compactness of cocycle associated with the system is proved by the tail-estimates method and splitting technique.

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1 Introduction

In this paper, we investigate the existence and uniqueness of the pullback random attractors for the following stochastic plate equations of Kirchhoff type with nonlinear damping on $\mathbb{R}^n$:

$$\begin{cases} u_{tt}+h(u_t)+\Delta^2 u+\lambda u-M(\|\nabla u\|^2)\Delta u+f(x,u)=g(x,t)+\phi(x)\dfrac{\mathrm{d}W}{\mathrm{d}t}, \\ u(x,\tau)=u_0(x),\ \ u_t(x,\tau)=u_1(x), \end{cases} \tag{1.1}$$

where $x\in\mathbb{R}^n$, $t>\tau$ with $\tau\in\mathbb{R}$, $\lambda>0$, $g(x,\cdot)$ and ϕ are given functions in $L^2_{loc}(\mathbb{R},H^1(\mathbb{R}^n))$ and $H^2(\mathbb{R}^n)\cap H^3(\mathbb{R}^n)$, respectively, $W(t)$ is a two-sided real-valued Wiener process on a probability space.

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For the nonlinear function $h(\cdot)$, we assume there exist two constants β_1,β_2 such that

$$h(0)=0, \quad 0<\beta_1\le h'(v)\le\beta_2<+\infty, \qquad \forall v\in\mathbb{R}. \tag{1.2}$$

The function $M(\cdot)$ satisfies the following conditions:
(1) $M\in C^1(\mathbb{R})$, such that

$$M_1\le M(s)\le M_2, \tag{1.3}$$

where M_1 and M_2 are some positive real constants;
(2) Let $\hat{M}(z)=\int_0^z M(r)\mathrm{d}r$, for $\forall\ z\ge 0$,

$$M(z)z\ge\hat{M}(z)\ge 0. \tag{1.4}$$

Furthermore, we assume that $f(x,\cdot)\in C^2(\mathbb{R})$, let $F(x,u)=\int_0^u f(x,s)\mathrm{d}s$ for $x\in\mathbb{R}^n$ and $u\in\mathbb{R}$, there exist positive constants $c_i(i=1,2,3)$, such that

$$|f(x,u)|\le c_1|u|^p+\eta_1(x), \qquad \eta_1\in L^2(\mathbb{R}^n), \tag{1.5}$$
$$f(x,u)u-c_2F(x,u)\ge\eta_2(x), \qquad \eta_2\in L^1(\mathbb{R}^n), \tag{1.6}$$
$$F(x,u)\ge c_3|u|^{p+1}-\eta_3(x), \qquad \eta_3\in L^1(\mathbb{R}^n), \tag{1.7}$$
$$\left|\frac{\partial f}{\partial u}(x,u)\right|\le\beta, \ \left|\frac{\partial f}{\partial x}(x,u)\right|\le\eta_4(x), \qquad \eta_4\in L^2(\mathbb{R}^n), \tag{1.8}$$

where $\beta>0$, $1\le p\le\frac{n+4}{n-4}$. Notice that (1.5) and (1.6) imply

$$F(x,u)\le c(|u|^2+|u|^{p+1}+\eta_1^2+\eta_2). \tag{1.9}$$

The Kirchhoff type term $M(\|\nabla u\|^2)\Delta u$ was firstly introduced by Kirchhoff [1] to describe small vibrations of an elastic stretched string. The bibliography of studies on the Kirchhoff type equations is plentiful (cf. Arosio and Garavaldi [2], Brito [3], etc.). The nonlinear damping term $h(u_t)$ is a function with damping, especially for stochastic plate equations like (1.1), when $h(u_t)=\acute{A}u_t$, have been used as models to study the phenomena of stochastic resonance in physics.

A lot of authors have proved the existence and uniqueness of global attractors for deterministic plate equations in [4–12]. For stochastic case, the existence and uniqueness of pullback random attractors for plate equations have been investigated in [13–15] on bounded domains, while in entire space $\mathbb{R}^n$, the authors obtained the long-time behavior of the problem in [16–21].

For the case $M(s)\equiv 0$, Eq. (1.1) becomes

$$u_{tt}+h(u_t)+\Delta^2u+\lambda u+f(x,u)=g(x,t)+\phi(x)\frac{\mathrm{d}W}{\mathrm{d}t}, \tag{1.10}$$