



2012 年班级大合照

一段精彩的数学之旅——介绍一个高中数学夏令营

A Wonderful Math Program for High School Students

励容达

PROMYS 成立于 1989 年,其创始人是一群曾经参加并受益于 Ross Program (一个历史更加悠久的数学培养计划)的数学家,其目的是培养和发现有才华而又好学的数学学生。二十多年里经过 PROMYS 训练而如今已达研究生年龄的学生中,约有 50% 已经获得或正在攻读博士学位,其中大多是在与数学有关的专业。他们中约有 100 位已成为大学教授,其中约有 70% 是数学教授,其余的分布在计算机、物理、化学、生物、医学、法律、哲学及其它领域。

2011 及 2012 年夏天,我有机会去参加了这项活动,非常喜欢。在这里我把我的所见、所闻、所感写出来,与各位爱好数学特别是数论的朋友们分享。

“少年科学家数学项目”(简称 PROMYS)是一项每年在波士顿举行、时长 6 周面向全球高中生的夏令营活动。该活动始于 1989 年,由“罗斯项目”(阿诺德·罗斯在 1957 年创立)发展而来。

The Program in Mathematics for Young Scientists (PROMYS) is a six-week long summer camp in Boston, available to high school students from around the world. It started in 1989 based on the Ross Program (founded by Arnold Ross in 1957).



骨干教师 (从左至右): Margy Baruch, Glenn Stevens (主讲教师), David Fried, and Steve Rosenberg

学习模式:

第一年参加的大约五十名新生将有机会学习数论的基础课程。这里的学习方式比较独特: 它是以习题集为主导, 每个课日 (即非周末) 都会布置一套习题并在下一个课日提交。一批来自美国名校 (如哈佛、麻省理工、普林斯顿) 数学专业的学生作为助教, 平均每个助教负责四名学生。新生们将他们一天中的大部分时间都花在了习题集上, 由助教批改。一个典型的问题集包括一段计算型的问题、一段严格证明型的问题、还有一段 PODASIPS (Prove or Disprove and Salvage if Possible, 证明或反证及补救), 另外还有一些自由探讨和查找资料的, 围绕这 5 方面问题总共大约给出 17 个题目。

每天上午, 知名的数论专家格伦·史蒂文斯 (Glenn Stevens) 教授都会进行九十分钟的授课, 内容大致涵盖了三天前布置的习题集。这样做的目的是让学生在听老师讲解前自己动手试一试。

头三个星期, 是训练一年级学生严格的数学证明与推理能力。先让他们把关于整数的公理压缩到简单几条, 然后再证明有一定水平的结论 (例如: 存在 x, y 使得 $ax+by=1$ 当且仅当 $\gcd(a, b)=1$, 从 $a|bc, \gcd(a, b)=1$ 可以推出 $a|c$, 整数的唯一分解, 等等)。在第三个星期五有一次中考, 包含 30 个问题, 每题 12 分。通常学生能拿到 130 分左右。

接下来的三个星期, 课程的内容会向不同方向展开: 不同于整数的其它环的性质 (整数添加 $\sqrt{-5}$ 所形成的环, 亦即 $\mathbb{Z}[\sqrt{-5}]$, 等等)、连分数、关于凸体内格点个数的闵可夫斯基定理、默比乌斯变换、二次互逆定律以及其它种种。在最后一个星期会有一次大考。

每年还会有 20 个左右的“老学员” (第二或第三次来的学生), 他们可以有较多的选择。每天一次的数论课他们

Content:

The 50 or so first-year students engage in learning about foundations of number theory. The format of this program is somewhat unique: Its dominant aspect is the problem sets, handed out each weekday and due the next weekday. Each student has a counselor, who is usually a student studying mathematics at some prestigious university (Harvard, MIT, Princeton, etc). Each counselor would be in charge of four students. The problem sets are marked by the counselors. The students spend most of their day working on the problem sets. A typical problem set would consist of a section on numericals, a section on rigor, a section of PODASIPS (Prove or Disprove and Salvage if Possible), exploration sections and possibly a reading search, totaling around 17 problems give or take 5 questions. There were 90-minute lectures every weekday by Professor Glenn Stevens, a well-known expert in number theory. The lectures are designed so as to approximately cover the content of the problem sets from three weekdays ago. This was to ensure that students would be able to have a go at the problems on their own first. During the first three weeks, first-year students develop their ability for mathematical rigor and proof by producing a reduced inventory of axioms of integers and progressing to prove statements up to certain level. (Examples: there exist x and y such that $ax+by=1$ iff $\gcd(a,b)=1$), ($a|bc$, $\gcd(a,b)=1$ implies $a|c$, unique prime factorization...). On the Friday of the third week, there is a midterm test, which consists of about 30 problems worth 12 marks each. A typical score is 130. In the final three weeks, the content of the course branches out, examining properties of other rings (integers adjoint root -5 , etc), continued fractions, Minkowski's theorem about convex, symmetric spaces centered in the origin of a lattice, Mobius inversion, quadratic reciprocity, and many other topics. There is a final exam in the last week.

The 20 or so returning students (those who have come back for a second or third year) have a number of options available to them. While they are required to attend the first-year lectures about number theory, they do not need to complete number theory problem sets. There are a number of courses available to returning students; these may or may not change each year. This year, the courses available, ranked from what was generally seen as 'easiest' to 'hardest', were Abstract Algebra (taught by Marjorie Baruch from Syracuse university), Geometry and Symmetry (Steven Rosenberg, Boston University), and the Analytic Class Number Formula (taught by Jared Weinstein, Boston University). The returning-student courses were different to the number theory courses in format, in that new concepts



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必须参加，只是不需要做习题。他们可以参加若干专题课，这些课程每年或许会有些变化。今年（2012 年）有三门专题课，大家普遍认为从“易”到“难”的依次是：抽象代数——雪城（Syracuse）大学的马乔里·巴鲁克（Marjorie Baruch）教授主讲，几何与对称——波士顿大学的斯蒂芬·罗森伯格（Steven Rosenberg）教授主讲，理想类数的解析公式——波士顿大学的杰瑞德·温斯坦（Jared Weinstein）教授主讲。

开给“老学员”的课，在授课方式上与基础数论课不同。每天安排的作业是与当天授课内容同步的，而不像基础数论课有 3 天的延时。上述三门课，每周分别布置 3 次、2 次和 1 次作业。今年我上了抽象代数和理想类数的解析公式课。

在抽象代数课中，我们从群的定义开始，逐步进展到凯莱（Cayley）定理与西罗（Sylow）定理。在理想类数的解析公式课中，我们从黎曼 zeta 函数开始，渐进到理解理想数、狄利克雷特征，并最终找到了各种二次域和分圆域的理想类数。

were usually introduced synchronously in lectures and the homework, rather than on a three-day delay as in the case of number theory. Homework was assigned three, two and one times per week in the respective courses. I attended Abstract Algebra and the Analytic Class Number Formula courses this year. In the Abstract Algebra course, we started with the definition of a Group and progress to topics about the level of Cayley's Theorem and Sylow's Theorem. In the Analytic Class Number Formula course, we began with the meaning of the Riemann Zeta function, progressed to start understanding ideals, looked at Dirichlet characters, and found the class number of various quadratic fields and cyclotomic fields.

In the midterm and final exam, if any returning students did particularly well in previous years, they would be allowed to take more difficult exams called, respectively, the 'short', the 'super short', and the 'duper super short' due to the fact that they