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Non-Local and Fully Connected Tensor Network Decomposition for Remote Sensing Image Denoising

Zhihui Tu[1], Shunda Chen[1], Jian Lu[1,2,3,*], Lin Li[4]
and Qingtang Jiang[5]

[1] *Shenzhen Key Laboratory of Advanced Machine Learning and Applications, School of Mathematical Sciences, Shenzhen University, Shenzhen 518060, China*
[2] *National Center for Applied Mathematics Shenzhen (NCAMS), Shenzhen 518055, China*
[3] *Pazhou Lab, Guangzhou 510320, China*
[4] *School of Electronic Engineering, Xidian University, Xi'an 710071, China*
[5] *Department of Mathematics and Statistics, University of Missouri-St. Louis, St. Louis, MO 63121, USA*

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Abstract. Remote sensing images (RSIs) encompass abundant spatial and spectral/temporal information, finding wide applications in various domains. However, during image acquisition and transmission, RSI often encounter noise interference, which adversely affects the accuracy of subsequent applications. To address this issue, this paper proposes a novel non-local fully connected tensor network (NLFCTN) decomposition algorithm for denoising RSI, aiming to fully exploit their global correlation and non-local self-similarity (NSS) characteristics. FCTN, as a recently developed tensor decomposition technique, exhibits remarkable capability in capturing global correlations and minimizing information loss. In addition, we introduce an efficient algorithm based on proximal alternating minimization (PAM) to efficiently solve the model and prove the convergence. The effectiveness of the proposed method is validated through denoising experiments on both simulated and real RSI data, employing objective evaluation metrics and subjective visual assessments. The results of the experiment show that the proposed method outperforms other RSI denoising techniques in terms of denoising performance.

AMS subject classifications: 65F22, 65F35, 94A08, 46N10
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1. Introduction

Recently, remote sensing images (RSIs) [10] have gained widespread utilization across various domains. However, in practical applications, RSIs inevitably suffer from

*Corresponding author. *Email address:* jianlu@szu.edu.cn (J. Lu)

noise contamination [17]. Noise presence markedly degrades image quality, consequently impacting subsequent tasks, including classification [26], sparse unmixing, target segmentation [25], and detection. Consequently, denoising emerges as an inevitable challenge during post-analysis and preprocessing of RSIs.

RSI exhibit rich spatial, spectral, or temporal features, which can be utilized to aid in RSI denoising. In general, RSI denoising involves leveraging the global correlations [23], non-local self-similarity (NSS) [5, 8], piecewise smoothness [7], and deep priors [22, 32] of the data. Due to the vast amount of data, RSIs often contain redundant information, namely global correlation. For instance, in hyperspectral images (HSIs), there is notable correlation among their spectral bands, resulting in the spectral vector residing in a lower-dimensional subspace [3]. Mathematically, we can characterize this global correlation using a low-rank representation, representing high-dimensional data based on learned lower-dimensional bases [34]. In the past two decades, low-rank matrix approximation methods have received significant attention, providing a theoretical foundation for RSI denoising and achieving promising results [15, 16, 29]. For instance, Zhang *et al.* [30] proposed a low-rank matrix recovery (LRMR) model, which achieved promising results by unfolding the RSI into a matrix. He *et al.* [12] introduced the total variation (TV) and proposed a low-rank matrix decomposition with total variation regularization (LRTV) method. However, matrix-based approaches often disrupt the intrinsic structure of high-order RSI data when unfolding it into matrices.

In recent years, tensor-based models have been proposed to recover RSI, drawing inspiration from the successful application of tensors in image processing inverse problems. These models enable effective exploration of the intrinsic properties of high-dimensional data. RSI data can be considered as tensors, where HSIs and multispectral images (MSIs) correspond to third-order tensors, while multi-temporal RSI are represented as fourth-order tensors. Liu *et al.* [14] used the PARAFAC model and statistical performance analysis to efficiently reconstruct the noisy RSI. Guo *et al.* [11] recovered RSI based on rank-1 tensor decomposition. Zhao *et al.* [35] proposed the constrained tube rank and sparsity model (CTSD) for addressing the problem of mixed noise removal in RSI. This model incorporates a low tube rank constraint, as well as ℓ_0 and ℓ_1 norm constraints, to effectively characterize the underlying clean RSI and sparse noise components. Zhuang *et al.* [39] proposed a global local factorization (GLF) method combining global matrix decomposition and local tensor decomposition, and achieved good results. Chen *et al.* [6] proposed a non-local group sparsifying transform learning (TLNLGS) method for HSI denoising. Nonetheless, t-SVD-based methods are limited to third-order tensors and lack flexibility in handling tensor correlation across different modes. Based on Tucker decomposition, Renard *et al.* [21] proposed a low-rank tensor approximation (LRTA) method, which effectively mines the low-rank attributes of different modes of data. However, the LRTA method only considers the global correlation of data, and cannot reconstruct the details and edge information of RSI well. To this end, Wang *et al.* [27] proposed a new method combining low-rank tensor decomposition and total variation (LRTDTV), which can better restore the RSI damaged