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Manifold Triangular Mesh Editing Based on Finite Differences and Fourier Series

Qunzhi Jin[1] and Yuanfeng Jin[1,2,*]

[1] *School of Mathematical Sciences, Harbin Engineering University, Harbin 150000, China*
[2] *College of Science, Yanbian University, Yanji 133002, China*

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Abstract. In this paper, the proposed method utilizes finite difference and Fourier series methods to calculate the mean curvature vectors and normalized curvature weights at the vertices of manifold triangular meshes. Specifically, this stable method achieves the L^2 convergence of the mean curvature vector. Furthermore, by comparing the method proposed in this paper with previously proposed classical methods, the results show that this method effectively balances precision and stability, and significantly reduces the larger errors observed on triangular meshes with small angles (approximately $0°$).

AMS subject classifications: 65M10, 78A48

Key words: Finite difference method, Fourier series, discrete curvature, small angles.

1. Introduction

The purpose of this paper is to integrate finite difference and Fourier series applications for manifold triangular mesh editing and to demonstrate that they can provide accurate results, while also verifying their improved stability by comparison with classical methods. On the manifold triangular mesh, this paper considers three issues:

1. Accurately calculate the mean curvature vector through finite difference.
2. Accurately calculate normalized curvature weights through Fourier series.
3. Use to solve numerical instabilities in the process of manifold triangular mesh editing.

*Corresponding author. *Email address:* `yfkim@ybu.edu.cn` (Y. Jin)

Finding a stable mean curvature vector on discrete surfaces plays an important role in computer graphics, computational geometry, and certain manifold triangular mesh editing problems [6]. One of the tools to study this issue is the Laplace-Beltrami operator, referred to in this paper as the LBO. The LBO can capture intrinsic topological and geometric information about spaced data. Consequently, it is utilized for achieving mesh smoothing [5] and facilitating mesh deformation [3]. Theoretically, this operator is a fundamental bridge linking analysis with differential geometry and physics, and many related achievements have already been made [8]. Meanwhile, by applying the Laplace-Beltrami operator (LBO) to the embedding of surfaces in R^3, various formulas have been proposed, as documented in various studies [13]. Moreover, without restrictive assumptions on the mesh, other standard discretization methods will also not converge [23]. This paper adopts the generalized difference method proposed in [10, 11], presents the piecewise linear finite difference approximation of the mean curvature vector, and combines it with Fourier series to provide convergence of piecewise linear surfaces in the L^2 norm.

In summary, within manifold triangular meshes, this paper designs an LBO discretization scheme using the differential schemes of triangular meshes and Fourier series to compute normalized curvature weights, which is applied to the editing of manifold triangular meshes. Furthermore, this paper provides corresponding mathematical analysis, offering theoretical assurance for the proposed discretization schemes. Moreover, the discrete LBO that combines finite differences and Fourier series is a suitable tool for solving the problem of editing manifold triangular meshes. The main contributions of this paper can be summarized as follows:

1. To solve for the mean curvature vector at the vertices of manifold triangular meshes, a differential schemes for triangular meshes is provided.
2. In the Laplace operator of manifold triangular meshes, by introducing Fourier series and analyzing its convergence, the normalized curvature weights at the vertices of triangular meshes are derived, providing a feasible solution to the numerical instability during the surface editing process.

The organization of the rest of this paper is as follows. In the next section, related work will be reviewed. In Section 3, some necessary background knowledge on manifold triangular mesh editing is provided, especially focusing on surface editing and the LBO. In Section 4, we provide the specific methods for finite difference schemes for triangular meshes and Fourier series normalized Laplace-Beltrami operator. Section 5 presents some experimental results. Finally, the conclusion of this article is presented.

2. Related work

In this section, the representative works on the discretization schemes of the LBO are revisited, and methods of manifold triangular mesh editing based on the Laplacian operator are reviewed, in order to facilitate independent work.