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Two Dynamical Models Based on Projection Operator for Solving the System of Absolute Value Equations Associated with Second-Order Cone

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Abstract. A new equivalent reformulation of the absolute value equations associated with second-order cone (SOCAVEs) is emphasised, from which two dynamical models based on projection operator for solving SOCAVEs are constructed. Under suitable conditions, it is proved that the equilibrium points of the dynamical systems exist and could be (globally) asymptotically stable. The effectiveness of the proposed methods are illustrated by some numerical simulations.

AMS subject classifications: 90C30, 90C33, 65K10
Key words: Absolute value equations, second-order cone, dynamical system, asymptotical stability, equilibrium point.

1. Introduction

The second-order cone (SOC) in $\mathbb{R}^n$ is defined by

$$\mathcal{K}^n = \left\{(x_1, x_2) \in \mathbb{R} \times \mathbb{R}^{n-1} : \|x_2\| \le x_1\right\},$$

where $\|\cdot\|$ denotes the Euclidean norm. If $n = 1$, let $\mathcal{K}^n$ represent the set of nonnegative reals. Moreover, a general SOC $\mathcal{K} \subset \mathbb{R}^n$ could be the Cartesian product of some SOCs [10, 11, 15], i.e.

$$\mathcal{K} = \mathcal{K}^{n_1} \times \cdots \times \mathcal{K}^{n_r}$$

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with $n_1, \cdots, n_r, r \geq 1$ and $n_1 + \cdots + n_r = n$. Without loss of generality, we focus on the case that $r = 1$ because all the analysis can be carried over to the setting of $r > 1$ according to the property of the Cartesian product. For any $x = (x_1, x_2) \in \mathbb{R} \times \mathbb{R}^{n-1}$ and $y = (y_1, y_2) \in \mathbb{R} \times \mathbb{R}^{n-1}$, their Jordan product is defined as [10, 11, 15]

$$x \circ y = (\langle x, y \rangle, y_1 x_2 + x_1 y_2) \in \mathbb{R} \times \mathbb{R}^{n-1},$$

where $\langle \cdot, \cdot \rangle$ denotes the Euclidean inner product in $\mathbb{R}^n$. With this definition, the absolute value vector $|x|$ in SOC $\mathcal{K}^n$ is computed by

$$|x| = \sqrt{x \circ x}. \tag{1.1}$$

In this paper, we consider the problem of solving the absolute value equations associated with SOC (SOCAVEs) of the form

$$Ax - |x| - b = 0 \tag{1.2}$$

with $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$. Unless otherwise stated, throughout this paper, $|x|$ is defined as in (1.1). SOCAVEs (1.2) is a special case of the generalized absolute value equations associated with SOC (SOCGAVEs)

$$Cx + D|x| - c = 0 \tag{1.3}$$

with $C, D \in \mathbb{R}^{m \times n}$ and $c \in \mathbb{R}^m$. To the best of our knowledge, SOCGAVEs (1.3) was formally introduced by Hu *et al.* [20] and further studied in [23, 39, 40, 42] and the references therein. In addition, SOCAVEs (1.2) is a natural extension of the standard absolute value equations (AVEs)

$$Ax - |x| = b, \tag{1.4}$$

meanwhile, SOCGAVEs (1.3) is an extension of the generalized absolute value equations (GAVEs)

$$Cx + D|x| = c. \tag{1.5}$$

In AVEs (1.4) and GAVEs (1.5), the vector $|x|$ denotes the componentwise absolute value of the vector $x \in \mathbb{R}^n$. It is known that GAVEs (1.5) with $m = n$ was first introduced by Rohn in [44] and further investigated in [18, 31, 43] and the references therein. Obviously, AVEs (1.4) is a special case of GAVEs (1.5).

Over the past two decades, AVEs (1.4) and GAVEs (1.5) have been widely studied because of their relevance to many mathematical programming problems, such as the linear complementarity problem (LCP), the bimatrix game and others, see e.g. [31, 34, 43]. Hence, abundant theoretical results and numerical algorithms for both AVEs (1.4) and GAVEs (1.5) have been established. On the theoretical aspect, for instance, Mangasarian [31] showed that solving GAVEs (1.5) is NP-hard; if GAVEs (1.5) has a solution, checking whether it has a unique solution or multiple solutions is NP-complete [43]. Moreover, various sufficient or necessary conditions on solvability and